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Towards maximal heat transfer rate densities for small-scale high effectiveness parallel-plate heat exchangers

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ABSTRACT

This paper addresses the question to what extent parallel-plate heat exchangers can be downsized without loss of thermal-hydraulic performance. It is shown that when the characteristic length scales of the channels are reduced at a constant pressure drop, the effectiveness exhibits a maximum due to axial heat conduction. The point of maximal effectiveness is found to correspond to a maximal thermal power density and thus to the minimal volume required for obtaining that effectiveness. Based on asymptotic relations for the effectiveness in the small and large channel limit, closed-form expressions are derived for the optimum geometric parameters that maximize power density in the limit of design effectiveness approaching unity. These relations are extended to a broader effectiveness range by means of dimensionless correction functions that are calculated numerically. The resulting expressions define optimal elemental units that can be used to construct parallel-plate counter-flow heat exchangers with the lowest possible core volume for effectiveness values between 0.53 and 1.

1. Introduction

With the advent of micro fabrication technology, small-scale heat exchangers are attracting increasingly more attention from research and industry. Due to their large surface area-to-volume ratio, a relatively high heat transfer rate can be obtained at a small size. This makes these components very promising for applications where thermal power density is an important factor like, for example, in small-scale energy conversion systems or in micro process engineering.

Thermal-hydraulic models for large-scale heat exchangers predict that thermal performance increases monotonically with decreasing dimensions of the flow passages. In reality however, upon reducing the length of the flow passages, at some point the thermal effectiveness will start to decrease due to axial heat conduction through the solid material between the fluid streams [1–4]. In view of maximizing energy recovery from the hot stream, it is clear that this effectiveness deterioration should be avoided. Therefore the question arises to what length scales a heat exchanger can be downsized while maintaining its thermal-hydraulic performance. The resulting design would have the smallest possible volume that accommodates a given heat transfer rate at a given effectiveness and pressure drop.

In literature, the problem of finding maximal heat transfer rate density of small-scale devices has been given most attention in electronics cooling applications. Bejan and Sciubba [5] investigated maximal power density design of a stack of heat generating boards of given length cooled by a fluid stream with specified pressure drop. The optimum board spacing was expressed in terms of a dimensionless pressure drop number, later denoted as the Bejan number [6]. It was concluded that, in order to achieve maximal heat transfer rate density, the thermal entrance length must be of the same order as the channel length. Later studies confirmed the optimal scaling law derived by Bejan and Sciubba numerically [7,8] and experimentally [9]. Moreover, other investigators successfully examined the theory for various other micro-channel geometries [10] and for heat sinks with conjugate heat transfer effects taken into account [8]. Although maximal power density is achieved, the optimization approach presented in the aforementioned references does not allow to choose the effectiveness independently. Moreover, since, by design, fluid packets in the center of the stream do not interact until the last part of the flow passage, there is little increase in fluid bulk temperature. Two-stream heat exchangers constructed from optimal cooling geometries will therefore have a rather low effectiveness. Thus, when, next to maximal power density, achieving a certain high thermal effectiveness value is a design requirement, another strategy must be adopted.

Several authors studied optimal design of small-scale heat exchangers with two fluid streams. Foli and co-workers [11] described the optimization of a micro channel heat exchanger using an analytical and CFD approach. In a first case, a fixed length and

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Nomenclature

Be	Bejan number, $\Delta PL^2/(\mu\alpha)$
C_1	constant defined in Eq. (12)
C_2	constant defined in Eq. (15)
c_p	fluid specific heat at constant pressure
D	plate spacing
$\widetilde{D}$	dimensionless plate spacing, D/L
h	mean convective heat transfer coefficient
H	total heat exchanger height
k	fluid conductivity
k_w	plate conductivity
$K(\infty)$	fully developed incremental pressure drop number
L	channel length
L^*	dimensionless thermal channel length, $L/(2DPe)$
L^*_{th}	dimensionless thermal entrance length
L^+	dimensionless hydraulic channel length, $L/(2DRe)$
L^+_{hy}	dimensionless hydraulic entrance length
M	axial conduction parameter, $UA'_{ax}/(\dot{m}'c_p)$
$\dot{m}'$	mass flow per unit length at each side
n	number of hot side or cold side channels
$N_H(\infty)$	fully developed incremental heat transfer number at constant heat flux
NTU	number of transfer units, $UA'/(\dot{m}'c_p)$
Nu	mean Nusselt number, $h2D/k$
f	Fanning friction factor
ΔP	pressure drop between inlet and outlet
Pe	Péclet number, $RePr$
Pr	Prandtl number, $c_p\mu/k$
$\dot{q}'$	transferred thermal power per unit length
$\mathcal{Q}$	thermal power density, $\dot{q}'/(HL)$
$Q^\diamond$	dimensionless thermal power density, $\mathcal{Q}\mu/(c_p\Delta T\rho\Delta P)$
Re	Reynolds number based on $2D$, $\rho U2D/\mu$
t	plate thickness
ΔT	hot to cold side inlet temperature difference
U	fluid bulk velocity in parallel-plate channel
UA'	hot to cold side conductance per unit length
UA'_{ax}	axial conductance per unit length
$W^\diamond$	dimensionless mass flow density, $\dot{m}'\mu/(HL\rho\Delta P)$

Greek symbols

α	fluid thermal diffusivity, $k/(\rho c_p)$
ε	heat exchanger effectiveness, $\dot{q}'/(\dot{m}'c_p\Delta T)$
μ	dynamic viscosity
ρ	fluid density
Φ	function used in effectiveness expression, Eq. (7)
Ψ	correction function defined in Eq. (35)
Ω	correction function defined in Eq. (34)

Subscripts

$*$	suboptimal value according to subproblem (21)
0	valid in the asymptotic case of $\widetilde{D} \to 0$
ai	result obtained from intersecting asymptotes
d	given design value
max	maximum
opt	optimum

cross-sectional area of the channels were considered leaving the aspect ratio as design variable. In a second case the volume of the channels was allowed to change by adapting the channel height. The results were presented in terms of trade-off curves between pressure drop and transferred thermal power. Lerou and co-workers [12] adopted an entropy minimization approach to the design of a micro channel heat exchanger used in a micro cryocooler system. They found optimal channel width, height and length for the specific system studied. Lee and co-workers [13] maximized the effectiveness of a micro channel cross-flow heat exchanger by combining finite element analyzes with a neuro-genetic optimization strategy. Pressure drop was not accounted for in this study. Bejan [14] proposed the conceptual design of a heat exchanger with dendritic counter-flows that connect elemental cross-flow constructs. The smaller-scale constructs were sized according to the maximal heat transfer rate density design theory. The larger-scale counter-flows allow for maximal flow access to the constructal elements and provide additional heat transfer. An experimental study of this concept by Raja et al. [15] showed effectiveness values of up to 74.12% which was significantly higher than the tested cross-flow designs with the same amount of heat transfer area.

This paper investigates at which length scale the power density of a heat exchanger is maximal for a given effectiveness and pressure drop. When the thermal power is fixed, the optimal design will have the smallest volume able to transfer that thermal power at the given effectiveness. For simplicity, the analysis is performed on a parallel-plate counter-flow heat exchanger geometry where the length and plate spacing are taken as design variables. An asymptotic method is adopted to solve the optimization problem approximately. This yields an analytic baseline solution that approaches the exact solution in the asymptotic case of high effectiveness. It is shown that the asymptotic solution and the exact solution differ only by a simple effectiveness dependent factor. Because of this, the exact solution of the design problem is easily retained as a correction to the asymptotic solution. The corresponding correction functions are correlated from numerical data. The resulting closed-form solution reveals the most important parameters that govern maximal power density.

The remainder of this text is structured as follows. In the next section the thermal-hydraulic heat exchanger model is presented. Subsequently, in Section 3, a dimensionless formulation of the problem is given and a simplified optimization approach is proposed. This approach is then developed analytically in Section 4 using an intersection of asymptotes method. This leads to closed-form expressions of the maximal power density design in the limit of high effectiveness. In Section 5, an exact semi-analytical solution is presented based on the previously obtained asymptotic result by introducing two dimensionless correction functions. The correction functions are calculated from a series of numerical optimizations and are subsequently correlated. Finally, in Section 6, the optimal designs are further analyzed in terms of their dimensionless flow and heat transfer characteristics and the validity of a fully developed flow and heat transfer assumption is discussed.

2. Mathematical model

As introduced in the previous section, this paper aims at a parallel-plate counter-flow heat exchanger optimization with respect to maximum heat transfer rate density for a preset effectiveness value. This will lead to expressions for the optimal plate spacing D and the optimal plate length L. For this optimization maximum allowable pressure drop ΔP, hot-to-cold side inlet temperature difference ΔT, relative plate thickness t/D as well as material and fluid properties are prescribed.

The considered geometry is depicted in Fig. 1. The heat exchanger consists of $2n+1$ plates of thickness t which form alternate flow passages for n hot and n cold side channels. Operation is assumed to be perfectly balanced with equal hot and cold side pressure drops, equal flow rates and equal temperature independent