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Transient double-diffusive convection in an enclosure with large density variations

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ABSTRACT

An extension of a slightly compressible flow model to double-diffusive convection of binary mixtures of ideal gases enclosed in a cavity is presented. The problem formulation is based on a low-Mach number approximation and the impermeable surface assumption is not invoked. The main objectives of this work are the statement of the mathematical model used, and the analysis of some significant results showing the influence of density variation on transient solutions for pure thermal or pure solutal convection as well as for thermosolutal convection in the special case where the thermal and solutal buoyancy forces are equal in intensity either for aiding or for opposing cases.

1. Introduction

Double-diffusive natural convection in enclosures filled with dilute binary mixtures has been investigated extensively both experimentally and numerically, especially for dilute liquid solutions which cover a large range of applications such as electrochemical and food processes, sensible heat storage in molten salt tanks, dispersion of contaminants in ocean flows, metallurgy and crystal growth from melts, biology, etc. Since the extensive review carried out by Viskanta et al. [29], the knowledge in double diffusive natural convection in liquids was thus considerably improved.

Natural convection in rectangular enclosures with combined horizontal temperature and concentration differences was studied experimentally by Kamotani et al. [15] and numerically by Lee and Hyun [20] for large Lewis numbers representative of water based solutions. Multicellular flow structures were also observed in an experimental study by Han and Kuehn [12] and simulated successfully [13]. The works on double-diffusive natural convection in confined binary gases are less. The explanation is that experiments are highly difficult to conduct. Simultaneous transfer of heat and moisture by natural convection in horizontal and vertical rectangular cavities of aspect ratio $A = 7$ representative of building structures was numerically and experimentally investigated by Wee et al. [33]. Numerical results for steady-state double-diffusion in a square cavity filled with air, submitted to either augmenting or opposing temperature and concentration buoyancy forces were presented by Béghein et al. [3]. The aim of the study was to investigate the effect of a pollutant source located on the hot or cold vertical walls on the fluid motion and heat and mass transfer rates. Transient natural convection in a binary mixture in square enclosures was numerically considered by Lin et al. [21]. The emphasis was put on the effects of the combined thermal and solutal buoyancy forces on temporal evolutions of the flow pattern and heat and mass transfer. Double-diffusive natural convection in a vertical stack of square cavities filled with moist air, with heat and mass horizontal diffusive walls, was studied numerically by Costa [6]. In all of these works, the Boussinesq approximation was invoked since only dilute gas mixtures were considered.

During the past two decades, an increasingly number of publications has dealt with applications in which natural convection occurs when characteristic temperature difference is comparable to the average temperature of the fluid. In these cases, the temperature gradients are sufficiently large to cause significant variations in the fluid properties with temperature, and for cavity flows, with pressure also. Paolucci [23] and Chenoweth and Paolucci [5] presented a low Mach number filtering approximation which decouples the density from the dynamic pressure. The system of conservation equation is complemented by the equation of state and laws for property variations with temperature. This low-Mach number approximation has been used recently in many numerical works on purely thermal convection (refer for example to the recent review by Darbandi and Hosseinizadeh [7]) and, also for solving supercritical flows for van der Waals' fluids (see for example, Accary et al. [1,2]).

Heat and mass transport phenomena in vapor crystal growth ampoules were numerically studied in detail [11] by using a

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Nomenclature

a	thermal diffusivity [$m^2\ s^{-1}$]
A	aspect ratio, $A = H/L$
C_p	specific heat [$J\ K^{-1}\ kg^{-1}$]
C_w	dimensionless wall-velocity parameter defined in Eq. (22)
$D_{1,2}$	binary mass diffusion coefficient [$m^2\ s^{-1}$]
Fr	Froude number, $Fr = u_d^2/gL$
g	gravitational acceleration [$m\ s^{-2}$]
H	cavity height [m]
$\bar{\bar{I}}$	unit tensor
$\vec{J}_i$	mass flux density through surface S_i [$kg\ m^{-2}\ s^{-1}$]
k_r	thermal conductivity [$W\ m^{-1}\ K^{-1}$]
L	cavity width [m]
Le	Lewis number, $Le = a/D_{1,2}$
$\dot{m}$	mass flow rate [$kg\ s^{-1}$]
M	molecular weight [kg/kmol]
M^*	molecular weight ratio, $M^* = M_2/M_1$
$\vec{n}_i$	unit outward normal to surface dS_i
Nu	local Nusselt number (Eq. (28))
N_x, N_y	numbers of grid points in x- and y-directions
p'_m	fluctuating part of the motion pressure [Pa]
$\bar{P}$	thermodynamic pressure [Pa]
Pr	mixture Prandtl number, $Pr = \mu_r/\rho_r a$
R	universal gas constant, $R = 8.315\ kJ\ kmol^{-1}$
Ra_m	solutal Rayleigh number, $Ra_m = \rho_r g \beta_M (W_w - W_0) L^3/a\mu_r$
Ra_T	thermal Rayleigh number, $Ra_T = \rho_r g \beta_T (T_w - T_0) L^3/a\mu_r$
Sc	Schmidt number, $Sc = \mu_r/\rho_r D_{1,2}$
Sh	local Sherwood number (Eq. (29))
u_d	thermal diffusion velocity, $u_d = a/L$ [$m\ s^{-1}$]
t	time [s]
T	temperature [K]
$\vec{V} = (u, v)$	velocity vector [ms^{-1}]
W	mass fraction
(x, y)	coordinates [m]

Greek symbols

β_M	solutal coefficient of volumetric expansion
β_T	thermal coefficient of volumetric expansion, $\beta_T = 1/T_r$ [K^{-1}]
ΔT	temperature difference, $\Delta T = (T_h - T_c)$ [K]
ΔW_2	mass fraction difference, $\Delta W_2 = (W_{2,h} - W_{2,c})$
ϵ_T	non-Boussinesq thermal parameter, $\epsilon_T = T_r/\Delta T$
ϵ_m	non-Boussinesq solutal parameter (Eq. (21))
γ	specific heat ratio
μ_r	mixture dynamic viscosity [$N\ m^{-1}\ s^{-1}$]
Ψ	streamfunction
ρ	density [$kg\ m^{-3}$]
τ	dimensionless time
$\bar{\bar{\tau}}$	viscous stress tensor
θ	dimensionless temperature ratio, $\theta = (T - T_r)/\Delta T$
ξ_x, ξ_y	stretching parameters in x- and y-directions

Subscripts

adv	advection
c	cold wall
$diff$	diffusion
h	hot wall
m	mixture
r	reference quantity
w	wall
0	quantity at initial state
1	refer to species "1"
2	refer to species "2"

Superscripts

–	average quantity
*	dimensionless quantity

compressible, elliptical form of the Navier–Stokes and species conservation equations in conjunction with the ideal gas law and Dalton's law, in order to link the temperature field (assumed to be prescribed) and concentration field within a confined binary mixture. Later on [22], the thermal effects were more accurately accounted for by solving the energy equation for a wide range of Grahsof number, but by assuming that the molecular weights of the two vapor components are equal. The effects of thermophysical property variations on the heat and mass transfer in a rectangular enclosure in which a mass flux occurs at the hot vertical wall due to sublimation of a solid or evaporation of a liquid and condensation at the opposite cold wall were considered by Weaver and Viskanta [30,31]. A porous-wall approach was used and the interdiffusion heat flux, Soret and Dufour effects were accounted for. Since mass conservation within the cavity was assumed, the condensable species was introduced into and removed from the cavity at the hot and cold walls, respectively. An experimental set-up was built to check the numerical predictions [32]. However, the comparisons showed only fair agreement for aiding flows and poor agreement for opposing flows.

This work addresses thermosolutal flows of binary ideal gas mixtures subjected to large temperature differences and/or large mass fraction differences (non-dilute binary mixture) rendering the Boussinesq and impermeable surface approximations inappropriate. These flows are however characterized by small Mach numbers, allowing not to use a purely compressible problem formulation.

2. Problem formulation

2.1. General formulation

The geometry investigated is an enclosure filled with a gas mixture in which natural convection is driven by thermal and solutal gradients. Distributions of temperature and mass fractions are specified at the cavity surface. The flow is assumed to be laminar, there are no chemical reactions, heat generation or heat dissipation. The mixture is radiatively transparent and the radiative exchanges amongst the surfaces are not accounted for. The heat flux driven by species interdiffusion and mass fraction gradients (Dufour effect) and, the mass flux driven by temperature gradients (Soret effect) are neglected. At initial state, the cavity is assumed to be filled with a mixture at uniform temperature and concentration fields.

The continuity equation for the mixture reads:

$$\frac{\partial \rho_m}{\partial t} + \nabla.(\rho_m \vec{V}) = 0 \quad (1)$$

where ρ_m is the mixture density. The momentum equation is written as

$$\frac{\partial (\rho_m \vec{V})}{\partial t} + \nabla.(\rho_m \vec{V} \otimes \vec{V}) = -\nabla p'_m + \nabla.\bar{\bar{\tau}} + (\rho_m - \rho_r)\vec{g} \quad (2)$$

where $\bar{\bar{\tau}}$ is the viscous stress tensor for a Newtonian fluid mixture written as