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Numerical simulation of natural convection heat transfer in a porous cavity heated from below using a non-Darcian and thermal non-equilibrium model

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Abstract

The present paper investigates the numerical simulation of steady laminar incompressible natural convection heat transfer in an enclosed cavity that is filled with a fluid-saturated porous medium. The bottom wall is subjected to a relatively higher temperature than the top wall while the vertical walls are considered to be insulated. The flow field is modeled upon incorporating different non-Darcian effects, such as the convective term, Brinkman effect and Forchhiemer quadratic inertial effect. Moreover the two-equation model is used to separately account for the local fluid and solid temperatures. The numerical solution is obtained through the application of the finite volume method. The appraisals of the sought objectives are performed upon identifying key dimensionless groups of parameters. These dimensionless groups along with their operating domains are: Rayleigh number $1 \leqslant Ra \leqslant 400$, Darcy number $10^{-4} \leqslant Da \leqslant 10^{-3}$, effective fluid-to-solid thermal conductivity ratio $0.1 \leqslant \kappa \leqslant 1.0$, and the modified Biot number $1 \leqslant \chi \leqslant 100$. The non-Darcian effects are first examined over a broad range of Rayleigh number. Next, the implications of the group of parameters on the flow circulation intensity, local thermal non-equilibrium (LTNE) and average Nusselt number are highlighted and pertinent observations are documented.

Keywords: Porous medium; Natural convection; Non-Darcy model; Local thermal non-equilibrium model; Numerical simulation

1. Introduction

Natural convection in a fluid-saturated porous medium is of significant interest to researchers owing to its various applications in different fields such as geothermal energy modeling, thermal insulation material, cooling of electronic devices and solar receivers to name a few. Several excellent monographs summarizing the state-of-the-art available in the literature testify to the maturity of this area, see for example, Nield and Bejan [1], Ingham and Pop [2], Vafai [3], Pop and Ingham [4], Bejan and Kraus [5], Ingham et al. [6] and Bejan et al. [7].

The buoyancy-driven convection associated with a cavity heated from below brings about patterns of convection cells. In each cell, the fluid rotates in a closed orbit and the direction of rotation alternates with successive cells. This phenomenon is conventionally referred to in the literature as the Bénard convection. Such a convection phenomenon also receives a broad attention owing to the inherited hydrodynamic fluid stability. The critical Rayleigh number, which signals the onset of natural convection, was first reported by Lapwood [8] to be equal to $4\pi^2$ for a Darcy fluid flow in a porous medium bounded between two infinite horizontal surfaces maintained at two different isothermal temperatures.

The presence of a porous medium inside the cavity hinders the buoyancy-driven activities. In essence, the momentum transport process in a porous medium is governed by several inherited phenomena such as the non-Darcian

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Nomenclature

A	aspect ratio, L/H
a_{sf}	specific surface area (m^2/m^3)
c_f	fluid specific heat (J/(kg K))
Da	Darcy number, K/H^2
F	inertia coefficient
h_{sf}	interstitial heat transfer coefficient ($W\,m^{-2}\,K^{-1}$)
H	cavity height (m)
k	thermal conductivity ($W\,m^{-1}\,K^{-1}$)
k_m	modified thermal conductivity, $\varepsilon k_f + (1-\varepsilon)k_s$
K	permeability (m^2)
L	cavity length (m)
P	dimensionless pressure, $pH^2/\rho_f\alpha_m^2$
p	pressure (kPa)
Pr	Prandtl number, ν_f/α_f
Ra	Rayleigh–Darcy number, $g\beta(T_h - T_c)H^3 Da/\nu_f\alpha_m$
T	intrinsic average fluid or solid temperature (K)
u, v	interstitial velocity components (m/s)
$\mathbf{v}$	interstitial velocity vector (m/s)
$\mathbf{V}$	dimensionless interstitial velocity vector, $\mathbf{v}H/\alpha_m$
U, V	dimensionless interstitial velocity components
x, y	spatial axial and transverse coordinates (m)
X	dimensionless axial coordinates, x/H
Y	dimensionless radial coordinates, y/H
α_f	thermal diffusivity $(k/\rho c)_f$
α_m	modified thermal diffusivity, $k_m/(\rho c)$
χ	modified Biot number, $h_{sf}a_{sf}H^2/k_m$
ε	porosity (m^3/m^3)
κ	effective fluid-to-solid thermal conductivity ratio, k_{feff}/k_{seff}
ν_f	fluid kinematic viscosity ($m^2\,s^{-1}$)
ρ_f	fluid density (kg/m^3)
θ	dimensionless temperature, $(T - T_c)/(T_h - T_c)$
Ψ	dimensionless stream function

Subscripts

c	cold
eff	effective
f	fluid
h	hot
s	solid

effects. These non-Darcian effects represent deviations from the familiar Darcy's law. Such non-Darcian effects include the viscous and quadratic inertial effects and the spatial-porosity variation effect. In addition, the modeling of the energy transport mechanism in a porous medium has its own share of challenging fronts. For example, the flow through the tortuous paths of a porous structure offers flow recirculation and mixing, which is classified in the literature under thermal dispersion effect. Moreover, modeling of a porous medium transport coefficient, i.e., energy carrier involves various presumptions and theories. Such challenges serve as ingredients for a wide debate and discussion over the appropriate modeling of the various effects, which is reflected in the large number of publications cited in the literature in this regard. The work of Kaviany [9], Nield and Bejan [1] and Vafai [3] can be cited as lead references in this regard.

The implications of the quadratic inertia term and the viscous term on natural convection heat transfer were tackled, for instance, by Chan et al. [10] and Lauriat and Prasad [11]. Also, the impact of Prandtl number on the Bénard convection was numerically investigated by Georgiadis and Catton [12] and Lage et al. [13]. Vasseur et al. [14] conducted a numerical simulation using the Darcy–Brinkman model to study the flow and thermal behaviors in a shallow cavity subjected to a uniform heating and cooling through opposite walls. Their results demonstrated the dependence of the Nusselt number predictions on the Darcy–Rayleigh number and Darcy number. Furthermore, Beji and Gobin [15] and Al-Amiri [16] have discussed the contribution of thermal dispersion to the overall natural convection heat transfer mechanism. Such an effect is customarily modeled as a diffusive term added to the effective thermal conductivity of the fluid phase. Both studies reported an appreciated increase in the computed Nusselt number upon incorporating thermal dispersion effect and better agreement with experimental results as well.

It is customary to handle the modeling of transport phenomena inside porous media using the volume-averaging method. The work of Vafai and Tien [17] is widely recognized for using the volume-averaging technique coupled with semi-empirical formulas to arrive at the two-dimensional momentum equation, which would complement the empirical energy conservation equation. The work of Khashan et al. [18] has elaborated on the implementation of the above method.

Our review of literature has indicated that most of the reported studies on Bénard convection had resorted to local thermal equilibrium (LTE) model, which presumes that the fluid and the solid phases are defined by a unique temperature at a given location within the porous medium. Such an assumption cannot be justified, however, when the temperature difference between the two phases is considered a crucial design parameter such as, for example, in porous metal heat exchangers and nuclear fluid rods placed in a coolant bath (see [19]). When the local fluid and solid phase temperatures are accounted for separately, two energy equations emerge to represent each phase. These equations supplemented with an additional term that models the modes of heat transfer between the two phases. In a series of studies spear-headed by Amiri and co-workers [20–22], the validity of local thermal equilibrium assump-