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Effect of optical properties on oscillatory hydromagnetic double-diffusive convection within semitransparent fluid

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Abstract

The effect of radiative heat transfer on the hydromagnetic double-diffusive convection in two-dimensional rectangular enclosure is studied numerically for fixed Prandtl, Rayleigh, and Lewis numbers, $Pr = 13.6$, $Ra = 10^5$, $Le = 2$. Uniform temperatures and concentrations are imposed along the vertical walls while the horizontal walls are assumed to be adiabatic and impermeable to mass transfer. The influences of the optical thickness and scattering albedo of the semitransparent fluid on heat and mass transfer with and without magnetic damping are depicted. When progressively varying the optical thickness, multiple solutions are obtained which are steady or oscillatory accordingly to the initial conditions. the mechanisms of the transitions between steady compositionally dominated flow and unsteady thermally dominated flow are analyzed.

Keywords: Hydromagnetic double-diffusive convection; Internal radiation; Radiative properties; Thermosolutal instability

1. Introduction

The double-diffusive convection, which takes place when compositionally driven buoyant convection and thermally driven buoyant convection occur simultaneously, arises in a very wide range of fields such as oceanography, astrophysics, chemical vapour transport process, drying process, crystal growth process, etc. This convection was widely experimentally and numerically studied for varied several non-dimensional parameters namely the Lewis and Prandtl numbers, the buoyancy ratio and for either aiding or opposing heat and mass gradients. Reviews on this subject can be found in the publications of Nishumira et al. [1], Ostrach [2], Viskanta et al. [3], Béghein et al. [4], Zhou and Zebib [5], Chamkha and Al-Naser [6], Costa [7,8] and Papanicalaou and Belessiotis [9]. It has been found that, for a buoyancy ratio close to the unity, an oscillatory flow caused by the interaction between thermal and compositional recirculations occurs. Nishumira et al. [1] carried out a careful depiction of the mechanism of this oscillatory flow. For the crystal growth process, the double-diffusive convection is induced because of the non-uniform distribution of impurities [10] and the quality of the growing crystal is severely affected by the melt convection and any oscillations are disadvantageous.

The request of an external magnetic field to control fluid flow and heat transfer in electrically conducting fluids has long been recognized in many applications such as crystal growth. Numerous studies on magnetoconvection were presented in the last few years especially for lower values of Prandtl number. Detailed bibliography can be found in Grandet et al. [11], Mößner and Müller [12] and Chamkha and Al-Naser [6]. Lately Aleksandrova and Molokov

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Nomenclature

B_0	magnetic induction
C	dimensionless species concentration, $C = (C' - C'_l)/(C'_h - C'_l)$
C_h	high species concentration
C_l	low species concentration
D	species diffusivity
g	acceleration of gravity
H	enclosure height
Ha	Hartmann number, $Ha = B_0 W\sqrt{\sigma_e/\mu}$
I	dimensionless radiant intensity, $I = I'/(n^2\sigma(T'_c)^4/\pi)$
I^0	dimensionless black body intensity, $I^0 = I^{0\prime}/(n^2\sigma(T'_c)^4/\pi)$
L	total number of discrete solid angles
L_+	total number of discrete solid angles oriented to a given boundary
Le	Lewis number, $Le = \alpha/D$
N	dimensionless quantity, $N_i^l = \frac{1}{\Delta\Omega^l}\int_{\Delta\Omega^l} \mathbf{\Omega n}_i\, d\Omega$
N	buoyancy ratio, $N = \beta_C(C'_h - C'_l)/\beta_T(T'_h - T'_c)$
n	refractive index
$\mathbf{n}$	unit vector normal to the control volume surface
Pr	Prandtl number, $Pr = \nu/\alpha$
Qc	dimensionless conductive heat flux averaged on isothermal walls
Qr	dimensionless radiative heat flux averaged on isothermal walls
Ra	Rayleigh number, $Ra = \beta_T g(T'_h - T'_c)W^3/(\nu\alpha)$
Rc	radiation conduction parameter, $\mathrm{Rc} = n^2 W T_c^3 \sigma/\lambda$
s	distance in the direction $\mathbf{\Omega}$ of the intensity
Sh	averaged Sherwood number defined in Eq. (14)
T	dimensionless temperature, $T = (T' - T'_c)/(T'_h - T'_c)$
t	dimensionless time, $t = \alpha t'/W^2$
T'_h	hot wall temperature
T'_c	cold wall temperature
u	dimensionless horizontal velocity, $u = u'W/\alpha$
v	dimensionless vertical velocity, $v = v'W/\alpha$
W	enclosure width
x, y	dimensionless Cartesian coordinates, $x = x'/W$, $y = y'/W$

Greek symbols

α	thermal diffusivity
β_T	coefficient of thermal expansion
β_C	coefficient of compositional expansion
β_r	extinction coefficient, $\beta_r = \sigma_r + \kappa$
ΔA	area of a control volume face
Δv	control volume
$\Delta\Omega$	control solid angle
ε_v	emissivity of vertical walls
ε_h	emissivity of horizontal walls
ϕ	temperature ratio, $\phi = T'_h/T'_c - 1$
κ	absorption coefficient
λ	thermal conductivity
μ	dynamic viscosity
ν	kinematic viscosity
σ	Stefan–Boltzmann constant
σ_e	electrical conductivity
σ_r	scattering coefficient
τ	optical width, $\tau = \beta_r W$
ω	dimensionless vorticity, $\omega = \omega' W^2/\alpha$
ω_0	scattering albedo, $\omega_0 = \sigma_r/\beta_r$
$\mathbf{\Omega}$	unit vector in the direction of the intensity
ψ	dimensionless stream function, $\psi = \psi'/\alpha$

Subscripts

e, w, n, s	faces of control volume centred in P
E, W, N, S	nodes around the nodal point P
P	nodal points

Superscripts

$'$	real variables
l, l'	discrete angular directions

[13] presented a three-dimensional study of buoyant convection in a rectangular cavity in presence of a strong magnetic field and Kenjeres and Hanjalic [14] considered magnetoconvection in fully turbulent regime and numerically studied, in addition to the magnetic damping, the laminarization effect of the external magnetic field. It has been found [12] that for a horizontal temperature gradient, a magnetic field perpendicular to the heated wall is most effective in damping the natural convection.

In crystal growth process of optical materials such as yttrium aluminum garnet (YAG), gadolinium gallium garnet (GGG) and lithium niobate (LN), the internal radiative heat transfer has a strong influence on the temperature distribution which affects the growing crystal quality. Taking the radiative heat transfer within both the crystal and the melt into account, Tsukada et al. [15] accomplished a global analysis of heat transfer for the crystal growth of an oxide ($LiNbO_3$). The authors used the differential approximation P1 to model radiative heat transfer in grey-emitting media. Their results show a noteworthy impact of optical properties of the crystal and the melt on flow and temperature fields. Especially, the melt–crystal interface shape becomes more convex toward the melt as the optical absorption coefficients of both the crystal and the melt decrease. Considering similar crystal, Kobayashi et al. [16,17] adopted the S4 discrete ordinates method to model