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Analytical solution for temperature profiles at the ends of thermal buffer tubes

Konstantin I. Matveev *, Gregory W. Swift, Scott Backhaus

Condensed Matter and Thermal Physics Group, Los Alamos National Laboratory, Los Alamos, NM 87545, United States

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Abstract

The distortion of temperature profiles at the ends of thermal buffer tubes is related to the time-dependent gas pressure and motion in both nearly adiabatic and nearly isothermal environments during one acoustic cycle. The analytical solution for the mean temperature distribution is derived assuming zero heat conduction between gas parcels and linear acoustics with the acoustic wavelength much longer than other system dimensions. Theoretical results are compared with some experimental data and with results of numerical simulations that assume high heat conductivity.

Keywords: Thermoacoustics; Thermal buffer tube; Pulse tube; Interface; Transition

1. Introduction

A thermal buffer tube (TBT) or a pulse tube sometimes separates two heat exchangers (HX) kept at different temperatures (Fig. 1(a)) inside thermoacoustic systems [1]. The role of this tube is to pass acoustic power but to minimize heat transfer between the heat exchangers. The inevitable heat leak along the tube depends in part on thermal boundary conditions at the tube ends. These conditions are defined not only by the heat exchanger temperatures but also by the acoustic field.

The details of the temperature profile at the end of such a tube at the interface with a heat exchanger were described and calculated numerically by Smith and Romm [2,3] in the context of thermodynamic irreversibility. Subsequent theoretical or numerical results were presented by Bauwens [4–7], de Boer [8], Kittel [9,10], Swift [1], Weiland and Zinn [11], and Matveev et al. [12]. Storch et al. [13] and Matveev et al. [12] reported experimental results supporting the theoretical understanding. However, none of these publications presented an analytical solution for the time-averaged temperature as a function of position in the region of interest.

The goal of this study is to derive an approximate analytical solution for the mean temperature profile in the same framework, assuming linear acoustics and negligible heat conduction. Such a solution will provide faster estimation of this effect without conducting numerical simulations and will give further insights on thermal phenomena inside TBTs.

2. Problem description and assumptions

A one-dimensional schematic of the problem is shown in Fig. 1(b). The working gas is ideal. One-dimensional linear acoustic oscillations (along the x-axis) are present in the system. The acoustic wavelength is much longer than any distance in this part of the system. Twice the acoustic displacement is shorter than the tube length. Viscosity and wall effects are neglected. No mean flow is present. Under

* Corresponding author. Present address: MME School, Washington State University, Pullman, WA, 99164-2920, United States. Tel.: +1 509 335 1327; fax: +1 509 335 4662.
E-mail address: matveev@hydrofoils.org (K.I. Matveev).

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Nomenclature

C	$\frac{\gamma-1}{\gamma}\frac{p_1}{p_m}$
p	pressure
T	Eulerian temperature
u	velocity
x	Eulerian coordinate
Y	defined by Eq. (12)

Greek symbols

φ	phase variable
γ	ratio of specific heats
θ	phase by which pressure oscillation leads velocity oscillation
Θ	Lagrangian temperature
ξ	Lagrangian coordinate
ψ	defined by Eq. (10)

Subscripts

HX	heat exchanger
m	mean
out	outside the doubled acoustic displacement
0	initial state
1	amplitude of sinusoidal motion
*	moment when gas parcel leaves heat exchanger

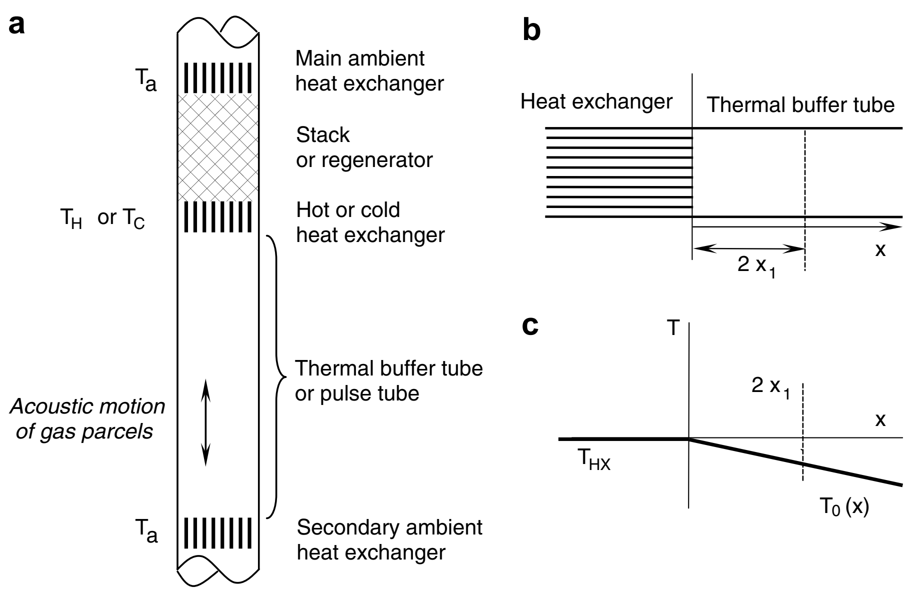

Fig. 1. (a) Typical part of a thermoacoustic system containing a thermal buffer tube or pulse tube. (b) Schematic drawing of the HX–TBT interface. (c) Example of a temperature profile at the beginning of an acoustic cycle.

these assumptions, a variation of the acoustic velocity amplitude, $\Delta u_1/u_1 \ll 1$, is small in the end zone $0 < x < 2x_1$, where $x = 0$ corresponds to the HX–TBT interface and x_1 is the acoustic displacement amplitude. This also implies relative smallness of the acoustic pressure amplitude, $2p_1/\gamma p_m \ll 1$. Therefore, all the gas parcels in the vicinity of the HX–TBT interface oscillate in phase, and the pressure fluctuation in the end zone is spatially uniform.

The environment is assumed isothermal inside the heat exchanger and nearly adiabatic inside the thermal buffer tube. A gas parcel entering the heat exchanger instantaneously acquires its temperature. A time-average linear temperature profile exists in the TBT beyond the doubled acoustic displacement, $2x_1$, from the HX–TBT interface. This temperature gradient implies a small but nonzero thermal conductivity of the gas. On the long time scale of many cycles, a small thermal conductivity can maintain a linear temperature profile for gas that never enters the heat exchanger, and on the time scale of one cycle such a small conductivity does not interfere with the evolving, continuous temperature profile of gas that has recently emerged from the heat exchanger. However, a small conductivity is not large enough to eliminate the temperature discontinuity at $x = 0$ that can occur when the gas reenters and strongly interacts with the isothermal heat exchanger.

Large temperature gradients are typically present in TBTs and pulse tubes. Cryogenic pulse tubes often span 200 K or more over a length of only 10 cm or less, with an average temperature of 200 K, with x_1 as large as 2 cm, and with $(\gamma - 1)p_1/\gamma p_m \sim 0.06$. Thus, in the following analysis we will retain terms of order $\frac{\gamma-1}{\gamma}\frac{p_1}{p_m}\frac{x_1}{T_m}\frac{dT_{m,out}}{dx}$, while neglecting terms of order $\left(\frac{\gamma-1}{\gamma}\frac{p_1}{p_m}\right)^2$.