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Buoyancy-driven convection of water near its density maximum with partially active vertical walls

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Abstract

Transient natural convection of cold water around its density maximum in a square cavity is studied numerically. Nine different positions of the active zones are considered. The governing equations are solved using Control volume method with power low scheme. The results obtained for various values of parameters are presented graphically in the form of streamlines and isotherms. It is found that the average Nusselt number behaves non-linearly as a function of Grashof number. The heat transfer rate is decreased in the density maximum regions.

Keywords: Transient convection; Partially active walls; Square cavity; Density maximum

1. Introduction

The buoyancy-driven convection in a fluid-filled cavity is a topic of interest for many researchers, due to its wide ranging of applications in cryogenic industry, cooling problems, crystal growth techniques, space applications, etc. Free convection arises in a fluid due to the density variations caused by the temperature differences of the system. In most of the analysis pertaining to the convection of water in enclosures, a linear temperature–density relationship was taken. But in practice this will never happen as the density of water varies with temperature in a nonlinear fashion, attaining its maximum density around 4 °C.

Ho and Tu [1] experimentally and numerically investigated the natural convection of water near its maximum density at high Rayleigh numbers. They observed oscillatory convection flow and temperature fields in the enclosure and provide a good agreement with the measured time period of the cyclic traveling wave motion of the maximum density contour. Kandaswamy and Kumar [2] studied the natural convection of water near its density maximum in the presence of uniform magnetic field. They observed that the effect of the magnetic field on the natural convection is to inhibit the heat transfer rate. The effect of density inversion on steady natural convection heat transfer of cold water is studied by Lin and Nansteel [3]. They found convection is reduced due to the density maximum.

Mahidjiba et al. [4] investigated onset of convection in a horizontal anisotropic porous layer saturated with water near 4 °C. It is found that the onset of motion dependent permeability ratio and inversion parameter. Michalek et al. [5] made a numerical benchmark study on natural convection for anomalous density variation of water, and compare performance of four different numerical methods. Convection in water above ice penetrates into the stably stratified region above the density maximum at 4 °C studied by Moore and Weiss [6]. They found steady convection occurs at Rayleigh numbers below the critical value predicted by linear theory. Pantokratoras [7] studied natural convection of water near the density extremum along a vertical plate with sinusoidal surface temperature variation. It is found that there is an inner boundary layer near the plate with periodic characteristics. Tong and Koster [10]

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numerically studied transient natural convection of water layer near its density maximum. The results illustrated that the temperature difference which determines the position of the maximum density plane in the water layer, can alter flow field and heat transfer substantially.

Valencia and Frederick [11] studied natural convection of air in square cavity with half-active and half insulated vertical wall. Recently Sundaravadivelu and Kandaswamy [9] derived a nonlinear 4th degree polynomial approximation for the density–temperature relation. In this paper we study natural convection of water near its density maximum in a square cavity of partially heating vertical walls using the above said nonlinear density–temperature relation.

2. Mathematical formulation

Consider a two-dimensional square cavity of size L filled with water as shown in Fig. 1. A portion of the left wall is kept at a temperature θ_h and a portion of the right wall is at temperature θ_c, with $\theta_h > \theta_c$, $\theta_c = 0\,^{\circ}\mathrm{C} = 273$ K. The remaining portion of the cavity is insulated. Nine different cases will be studied here. That is, the hot region is located at the top, middle and bottom and the cold region is moving from bottom to top of their respective walls. The density of water behaves non-linearly as $\rho = \rho_0\left[1-\sum_{i=1}^{4}(-1)^i\beta_i(\theta-\theta_c)^i\right]$ with $\beta_1 = 6.8143\times10^{-5}$, $\beta_2 = 9.9901\times10^{-6}$, $\beta_3 = 2.7217\times10^{-7}$ and $\beta_4 = 6.7252\times10^{-9}$. A typical density plot is provided in Fig. 3. It is clearly seen from the figure our 4th order polynomial is agreed with universal data. Under these assumptions the equations governing the motion of a two-dimensional viscous incompressible fluid may be written in the vorticity–stream function formulation as

$$\frac{\partial\zeta}{\partial\tau} + U\frac{\partial\zeta}{\partial X} + V\frac{\partial\zeta}{\partial Y} = \nabla^2\zeta + \sum_{i=1}^{4} i(-1)^i Gr_i T^{(i-1)}\frac{\partial T}{\partial Y} \tag{1}$$

$$\nabla^2\Psi = -\zeta \tag{2}$$

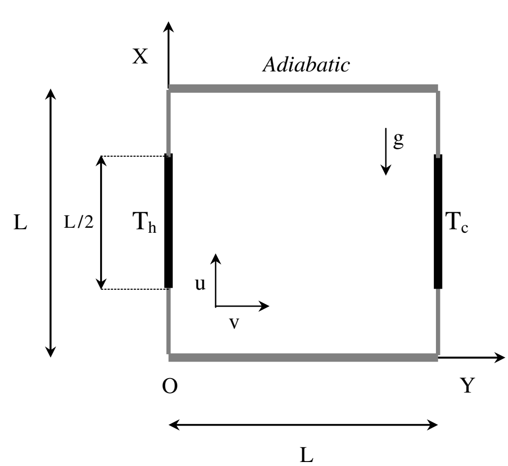

Fig. 1. Physical configuration.

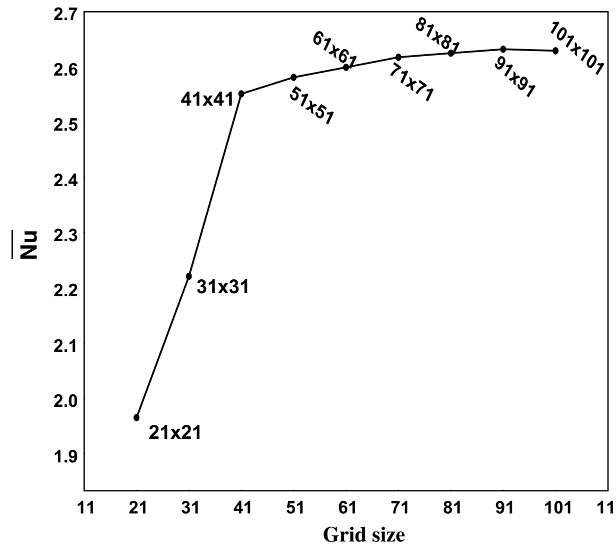

Fig. 2. Average Nusselt number versus different grid sizes for middle–middle heating location and $Gr_1 = 180{,}662$.

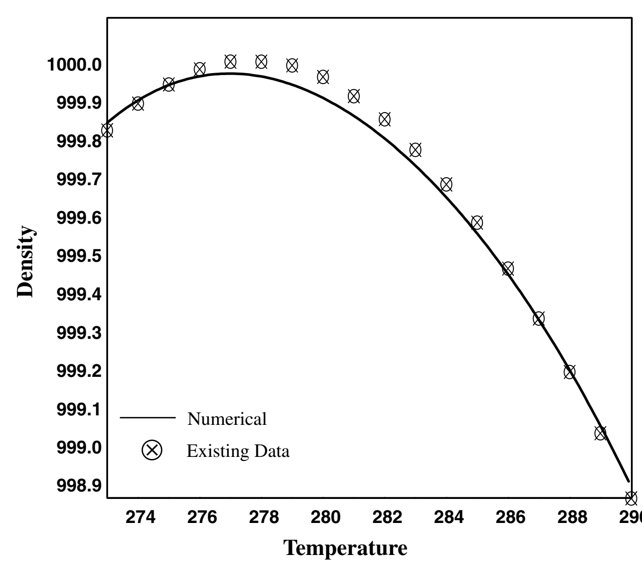

Fig. 3. Density versus temperature.

$$\frac{\partial T}{\partial\tau} + U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \frac{1}{Pr}\nabla^2 T \tag{3}$$

where

$$U = -\frac{\partial\Psi}{\partial Y}, \quad V = \frac{\partial\Psi}{\partial X} \quad \text{and} \quad \zeta = \frac{\partial U}{\partial Y} - \frac{\partial V}{\partial X}. \tag{4}$$

The initial and boundary conditions in the dimensionless form are:

$$\begin{array}{llll}
\tau = 0; & \Psi = 0 & T = 0 & \\
\tau > 0; & \Psi = \frac{\partial\Psi}{\partial X} = 0 & \frac{\partial T}{\partial X} = 0 & \text{at } X = 0 \text{ and } 1 \\
 & T = 1 & \text{hot part,} & T = 0 \text{ cold part at } Y = 0 \text{ and } 1 \\
 & \frac{\partial T}{\partial n} = 0 & \text{elsewhere} & \text{at } Y = 0 \text{ and } 1
\end{array}$$

The nondimensional variables are $\tau = \frac{t}{L^2/\nu}$, $(X, Y) = \frac{(x,y)}{L}$, $(U, V) = \frac{(u,v)}{\nu/L}$, $\Psi = \frac{\psi}{\nu}$, $\zeta = \frac{\omega}{\nu/L^2}$, $T = \frac{\theta-\theta_c}{\theta_h-\theta_c}$.