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Heat transfer with laminar forced convection in a porous channel exposed to a thermal asymmetry

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Abstract

The effect of thermal asymmetry on laminar forced convection heat transfer in a plane porous channel with Darcy dissipation has been investigated numerically. The parallel plates making the channel boundaries were kept at constant, but different temperatures. The thermal asymmetry thus imposed on the system, results in an asymmetric temperature field and different heat fluxes across the channel boundaries. Depending on Darcy, Peclét and Reynolds number, the thermal asymmetry may lead to a reversal of the heat flux at a certain position along the flow at least at one of the channel walls. The corresponding Nusselt numbers become zero and might experience discontinuities thereby jumping from infinite positive to infinite negative, or vice versa. This feature is observed not only in the region of thermal development, but also in the fully developed region. In the fully developed region, an analytical expressions for the Nusselt numbers were obtained. From these expressions, analytical equations were deduced for the calculations of the axial positions along the channel where the Nusselt numbers become zero, or experiences discontinuity.

Keywords: Porous media; Laminar developing flow; Heat transfer; Thermal asymmetry

1. Introduction

The heat transfer with forced convection in porous media is an interesting and challenging physical problem, the solution of which is important in several areas of engineering practice, see e.g. Bejan et al. [1]. It has, therefore, extensively been studied in the past, and various fluid flow and heat transfer arrangements have been treated both analytically and numerically, see e.g. Kaviany [2], Nield and Bejan [3], Bejan [4] and Vafai [5]. However, the problem is far from being completely solved, even the governing equations are still the subject matter of scientific debates, see e.g. Travkin and Catton [6], Gray and Miller [7], Bear and Bachmat [8], and Whitaker [9]. Nevertheless, the mathematical models used so far account for different effects and the solutions obtained are adapted to various boundary conditions. For instance, Kaviany [10] studied laminar forced convection in a porous channel bounded by isothermal parallel plates adopting the Brinkman-extended Darcy model. Vafai and Kim [11] arrived at a closed form solution with fully developed forced convection in a porous plane channel exposed to a symmetric heating at constant heat flux. Nield et al. [12] analysed the fully developed forced convection in a fluid-saturated porous-medium channel with isothermal or isoflux boundaries. Nield et al. [13] investigated the heat transfer in a thermally developing region of a hydrodynamically developed flow in a plane porous channel bounded by isothermal plates. The energy equation they used accounts for viscous dissipation and axial heat conduction. The solutions reported illustrate the effects of Brinkman, Peclét and Darcy numbers on the heat transfer for different dissipation models. Mohamad [14] investigated the flow field and heat transfer with laminar forced convection in conduits filled with a porous material to different degrees. As far as the homogeneously filled channel is concerned, the effect of Darcy number on heat transfer in the fully developed flow region may largely

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Nomenclature

c_p	specific heat at constant P
CP	cold plate
C_1	constant
C_2	constant
Da	Darcy number, Eq. (6)
Ec	Eckert number, Eq. (6)
HP	hot plate
k	thermal conductivity
K	permeability of porous medium
m	inverse square root of Da, Eq. (9)
M	viscosity ratio, Eq. (9)
Nu	Nusselt number, Eqs. (23) and (24)
p	dimensionless pressure, Eq. (6)
P	pressure
Pe	Peclét number, Eq. (6)
Pr	Prandtl number
q	heat flux
Re	Reynolds number, Eq. (6)
T	temperature
u	dimensionless velocity
U	velocity
W	half width of channel
x	dimensionless axial coordinate
X	axial coordinate
y	dimensionless cross coordinate
Y	cross coordinate
ε	heat source strength, Eq. (3)
σ	pressure drop function, Eq. (6)
κ	thermal diffusivity
ν	kinematic viscosity
μ	dynamic viscosity
θ	dimensionless temperature with thermal asymmetry, Eq. (6)
Θ	dimensionless temperature with thermal symmetry, Eq. (64)
ρ	density
Ω	parameter, Eqs. (17) and (64)
ξ	dimensionless axial coordinate, Eq. (18)
ψ	scaled temperature difference with thermal asymmetry, Eq. (18)
Ψ	scaled temperature difference with thermal symmetry, Eq. (64)

Indices

eff	effective property
C	cold
H	hot
IN	inlet
m	average
SL	slip flow
W	wall

be neglected for $Da > 1$. Haji-Sheikh and Vafai [15] performed heat transfer analysis on various cross sections of the conduits without a heat source giving detailed insights into the effect of Darcy number on thermal performance of porous inserts. For this model, Haji-Shekh [16] provided approximate expression for the Nusselt number.

When treating transport processes in porous media the so-called *Local Thermal Equilibrium* model (LTE) is frequently adopted. By this model, the fluid and the porous medium are considered as a single phase having physical properties of the actual phases mostly weighted by the volume fractions occupied by these phases. The applicability of this model is confined to a certain range of process and system parameters like fluid velocity and transport properties of the phases. Contrary to this model, the model of *Local Thermal Non-Equilibrium* (LTNE) accounts for thermal interaction among the phases within the porous system [17–23]. By this model the thermal interaction is based on a heat transfer coefficient at the phase interface within the porous system which is previously unknown. The LTNE two equations model is usually considered to be more adequate than the one equation LTE model. The boundary between these models regarding their applicability has been discussed in several papers, see e.g. Kim and Jang [19] and the references therein.

In summary, the treatments in all of the above mentioned references are restricted to thermally symmetric boundary conditions. In practice, however, it is indeed almost impossible to accomplish such conditions, and thermal asymmetry will probably be the rule rather than exception. Thermal asymmetry is shown by Mitrovic and Maletic [24] to materially affect the heat transfer in laminar forced convection in a conduit of annular cross section without a porous insert, and similar effects of thermal asymmetry may also be expected in the case of porous channels. Mahmud and Fraser [25] studied the heat transfer and entropy generation with laminar fully developed flow in a porous channel bounded by parallel plates, which were kept isothermal at different temperatures thus imposing on the system a thermal asymmetry. The inertia in the momentum equation was disregarded, while a volumetric source term was included into the energy equation. The physical properties were taken as constant, so the flow field is decoupled from the temperature field, and the one-dimensional transport equations were solved analytically. The thermal asymmetry results in an asymmetric temperature distribution in the porous gap the shape of which depends on the Eckert, the Prandtl and the Darcy number. Consequently, it also affects the heat transfer across the porous insert. The analysis of Mahmud and Fraser [25] is