International Journal of Heat and Mass Transfer 52 (2009) 4254–4263

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

On the onset of natural convection in differentially heated shallow fluid layers with internal heat generation

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ARTICLE INFO

Article history:
Received 11 June 2008
Received in revised form 1 January 2009
Accepted 3 April 2009
Available online 1 June 2009

Keywords:
Rayleigh Benard convection
Internal heat generation
Linear stability analysis
Stability curves
Critical Rayleigh number
Numerical investigation

ABSTRACT

This paper reports the results of an analytical and numerical investigation to determine the effect of internal heat generation on the onset of convection, in a differentially heated shallow fluid layer. The case with the bottom plate at a temperature higher than the top plate mimics the classical Rayleigh Benard convection. However, internal heat generation adds a new dimension to the problem. Linear stability analysis is first carried out for the case of an infinitely wide cavity. The effect of aspect ratio on the onset of convection is studied by solving the full Navier–Stokes equations and the equation of energy and observing the temperature contours. A bisection algorithm is used for an accurate prediction of the onset. The numerical results are used to plot the stability curves for eight different aspect ratios. A general correlation is developed to determine the onset of convection in a differentially heated cavity for various aspect ratios. For an aspect ratio of 10, it is seen that the cavity approaches the limit of an infinite cavity. Analytical results obtained by using linear stability analysis agree very well with the "full" CFD simulations, for the above aspect ratio.

1. Introduction

Natural convection in a shallow horizontal fluid layer with internal heat generation is gaining immense interest in contemporary heat transfer research, as it has a number of applications related to atmospheric sciences, as for example, in convection from the earth's mantle, convection in the outer layer of sun and stars, as discussed below. The convection in the earth's atmosphere can be modeled using the Rayleigh Benard convection as the temperature of the lower layers is higher than that of the upper layers. The heat transfer from the clouds and the associated phase change due to the condensation/evaporation of water vapor/liquid water and melting/freezing of ice/liquid water can be modeled using internal heat generation. The Meso scale Cellular Convection (MCC) is usually considered as an atmospheric manifestation of Rayleigh Benard convection (Agee et al. [1], Rothermel and Agee [2]). The prediction of the stability of a fluid layer in the atmosphere is crucial for meteorological studies. Similarly, the study of convection in the earth's mantle is important for geologists. The radiogenic heating together with thermal convection provides the driving force for any convection present in this stage of the Earth's history (see for example Tozer [3]). Nuclear fusion reaction causes volumetric heat generation in stellar interiors. The study of convection in the stars helps in obtaining information about the history of stars and various other astronomical events (Bodenschatz et al. [4]). The study of stability in a confined fluid too has a lot of applications. The heat transfer rate in the confined fluid can be increased by inducing flow. The cooling of the forward section of missiles and reentry vehicles are enhanced by induced flow. Stability analysis also finds applications in atomic power plants where a high rate of heat transfer in confined fluids is required.

A large number of investigations have been carried out on natural convection heat transfer that occurs in an enclosure due to a temperature difference across the enclosure. Most of the early investigations of this problem were based on the classical Rayleigh Benard convection that occurs in a fluid layer which is confined between two thermally conducting plates, and is heated from below to produce a fixed temperature difference (Fig. 1). Rayleigh Benard convection was first studied analytically by Lord Rayleigh in 1916 in relation to the experiments made by Benard in 1900 [5].

The present study is concerned with determining the stability of Rayleigh Benard convection with internal heat generation. Several well established methods are available in literature to determine the critical Rayleigh number for the onset of convection. The simplest approach is to numerically simulate the convection in the steady state starting with a Rayleigh number range across which the transition occurs, and using a bisection algorithm to detect the critical Rayleigh number. The onset of convection is obtained by observing the temperature contours. In the conduction regime, the temperature profiles are linear, while nonlinear temperature profiles are obtained for convection. Xia and Murthy [6] used this approach to investigate the flow transitions in deep three dimensional cavities heated from below, a configuration similar to the

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doi:10.1016/j.ijheatmasstransfer.2009.04.006

Nomenclature

AR	aspect ratio of cavity, L/H
C_p	specific heat of fluid, J/kgK
g	acceleration due to gravitational, 9.8 m/s^2
H	characteristic height of the domain, m
$\hat{i}$	unit vector in x direction
$\hat{j}$	unit vector in y direction
$\hat{k}$	unit vector in z direction
K	wave number
k	thermal conductivity, W/m K
L	characteristic length of the domain, m
Nu	Nusselt number, $\frac{q_w}{(T_{\max}-T_w)}\frac{H}{k}$
Pr	Prandtl number of fluid (air), ν/α
Ra	external Rayleigh number, $\frac{g\beta\Delta TH^3}{\upsilon\alpha}$
Ra_i	internal Rayleigh number, $\frac{g\beta q'''H^5}{64\upsilon\alpha k}$
Ra^*	nondimensional heat generation, $64Ra_i$
q'''	heat generation per unit volume, W/m^3
s	growth rate
t	time, s
T	temperature of fluid, K
T_C	temperature of top plate, K
T_H	temperature of bottom plate, K
T_o	reference temperature, K
T_i	dimensionless temperature difference $\frac{k\Delta T}{q'''H^2}$
U	velocity vector
u	velocity in x direction, m/s
v	velocity in y direction, m/s
w	velocity in z direction, m/s
W	amplitude of velocity perturbation in the z direction, m/s

Greek symbols

α	thermal diffusivity, m^2/s
β	coefficient of thermal expansion, 1/K
δ_{ij}	Kronecker's delta
Θ	amplitude of temperature perturbation
κ	wavenumber in x direction
λ	wavenumber in y direction
ν	kinematic viscosity, m^2/s
ρ	density of fluid, kg/m^3
ρ_o	density of fluid at reference temperature, kg/m^3

Rayleigh Benard convection. Here, the critical Rayleigh number for the onset of convection and the transition to turbulence were studied in tall cavities.

An extrapolation of the correlation between Nusselt number and Rayleigh number obtained either using experiments or numerical simulations to the conduction value of Nusselt number also yields the critical Rayleigh number for the onset of convection. This method gives a crude approximation and can be used only for one parameter problems. Kulacki and Goldstein [7] studied, experimentally, thermal convection in a horizontal fluid layer with uniform volumetric energy sources and the results were extrapolated to obtain the critical Rayleigh number for the onset of convection.

Linear stability analysis is another method dealt with in good detail in literature ([8,9]). In linear stability analysis, the effects of small disturbances on the system are studied. The system is said to be stable if the disturbances die out in time. The nonlinear terms in the governing equations are neglected as they are insignificant during the onset. The linearized conservation equations for the disturbance quantities are solved using suitable numerical techniques to determine the critical Rayleigh number for the onset of convection. Pellew and Southwell [10] were the first to present a rigorous proof for the principle of exchange of stability. The critical Rayleigh number for the onset of Rayleigh Benard convection was established as 1708 for the rigid–rigid configuration. Ostrach and Pnueli [11] describe a method to obtain upper bounds to the instability criterion for some particular configurations. Roberts [12] carried out a linear stability analysis for the onset of convection in a horizontal fluid layer with the bottom and top plates at the same temperature and with only internal heat generation driving the convection. Tasaka and Takeda [13] studied the effects of internal heat generation with bottom wall heating on natural convection in cavities using linear stability analysis.

If the disturbances are of sufficient magnitude, the nonlinear terms in the disturbance quantities must be retained in the conservation equation describing the altered fluid motion. The deduction of the critical Rayleigh number based on the complete nonlinear equations is known as energy theory [14] and will give a stability criterion which is usually more restrictive than that of the linear theory. Fusegi et al. [15] considered a square cavity with differentially heated walls, along with volumetric heat generation, using numerical methods. They defined two Rayleigh numbers: one based on the temperature difference between the walls, and the other based on heat sources in the cavity. The basic interest of the study was to determine the effect of these Rayleigh numbers on the flow field.

From the review of literature presented above, it is seen that Rayleigh Benard convection and its applications have been extensively documented. The fundamental principles required for linear stability analysis of Rayleigh Benard convection are also well established. Numerical and experimental studies on convection in a fluid layer with only internal heat generation are also available in literature. The linear stability analysis for the onset of convection in fluid layer with internal heat generation has also been discussed in literature. Even so, scarce are studies that address the effect of internal heat generation on the onset of Rayleigh Benard convection in spite of its potential applications in several problems in science and engi-

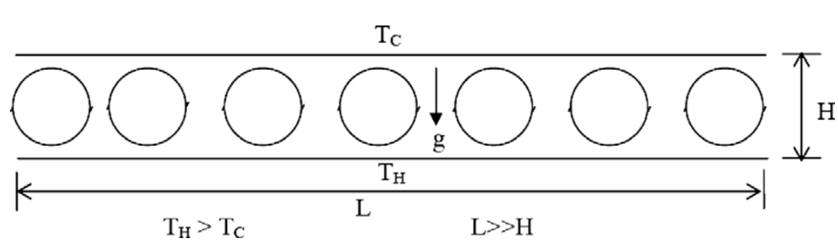

Fig. 1. Schematic of Rayleigh Benard convection.