Available online at www.sciencedirect.com
ScienceDirect
International Journal of Heat and Mass Transfer 51 (2008) 1034–1048
International Journal of HEAT and MASS TRANSFER
www.elsevier.com/locate/ijhmt

Mixed convection from an isolated spherical particle

S. Bhattacharyya *, A. Singh

Department of Mathematics, Indian Institute of Technology, Kharagpur 721 302, India

Received 3 May 2007; received in revised form 21 May 2007
Available online 19 December 2007

Abstract

A numerical study on mixed convection around a hot spherical particle moving vertically downwards in a still fluid medium has been made. The flow field is considered to be axisymmetric for the range of Reynolds number (based on the diameter and the settling velocity of the particle) considered. A third-order accurate upwind scheme is employed to compute the flow field and the temperature distribution. The form of the wake and the thermal field is analyzed for several values of Grashof number and the Reynolds number. The influence of buoyancy on drag and the rate of heat transfer are studied. At moderate Reynolds number, recirculating eddy develops in the downstream of the sphere. With the rise of surface temperature this eddy collapses and the fluid adjacent to the heated surface develops into a buoyant plume above the sphere. The increase in surface temperature of the sphere delays the flow separation. Our results show that the drag force and the rate of heat transfer strongly depend on Grashof number for the moderate values of Reynolds number. The conjugate heat transfer from the moving sphere is also addressed in the present paper. We have compared our computed solution with several empirical and asymptotic expressions available in the literature and found them in good agreement.

Keywords: Heat transfer; Thermal plume; Buoyancy; Upwind scheme; Separation

1. Introduction

The dynamics of solid particles submerged in a fluid medium is of interest in many engineering processes, such as vaporization and condensation of fuel droplets, manufacturing systems, fuel spray, coal combustion and motion of aerosol particles. The hydrodynamic interaction between suspended particles and the surrounding fluid phase is of interest also in colloid, polymer, aerosol and physiological systems. Over the years, several studies have been made on flow past particles of various shapes, such as rigid sphere, spheroids, and cylinders. A wide variety of numerical and analytical methods have been used to obtain solutions for a broad range of geometric and flow parameters. Most of the earlier studies have been included in the books by Happel and Brenner [1] and Clift et al. [2]. Recent works on numerical and experimental studies on spherical particles falling in unconfined fluids is discussed in Michaelides [3]. Several authors have used the finite element/ finite volume method for direct numerical simulation of particles sedimentation in 2D and 3D [4,5]. The particle–fluid interaction problems with high number of particles are complicated owing to the necessity of geometrically adapted grid generation. The lattice Boltzmann method (LBM) to simulate the particulate motion have proven to be more efficient in recent years. A detailed discussions on recent models using LBM have been made by Feng and Michaelides [6]. In this paper, however, we restricted our discussion on the motion due to a single particle.

The fluid flow past a stationary isolated sphere at varying Reynolds number has been considered by several authors experimentally and/or numerically because of its complex nature. The dye visualization study of Magavey and Bishop [7] reveals that sphere wake remains steady symmetric up to Reynolds number 210. Their results show that the transition from a steady axisymmetric with a

* Corresponding author.
E-mail address: somnath@maths.iitkgp.ernet.in (S. Bhattacharyya).

0017-9310/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijheatmasstransfer.2007.05.033

Nomenclature

C_d	drag coefficient, $F_d/\frac{1}{2}\rho U_\infty^2 \pi R^2$
c_p	specific heat
C_p	pressure coefficient, $(p^* - p_\infty)/\frac{1}{2}\rho U_\infty^2$
g	gravitational acceleration
Gr	Grashof number, $4g\beta(T_s - T_\infty)R^3/\nu^2$
$\overline{Nu}$	total Nusselt number
Nu	local Nusselt number
Pe	Peclet number, $2U_\infty R/\alpha$
Pr	Prandtl number, ν/α
r	dimensionless radial coordinate
R	sphere radius
Re	Reynolds number, $2U_\infty R/\nu$
Ri	Richardson number, Gr/Re^2
t	dimensionless time
T	dimensionless temperature
u	dimensionless radial velocity
U_∞	free stream velocity
v	dimensionless cross-radial velocity

Greek symbols

α	thermal diffusivity, $\kappa/\rho c_p$
α_e	thermal diffusivity of the solid sphere
β	coefficient of thermal expansion, $-(\frac{\partial\rho}{\partial T})_p/\rho_\infty$
κ	thermal conductivity
μ	dynamic viscosity
ν	kinematic viscosity, μ/ρ
ρ	fluid density
θ	angular coordinate
ψ	non-dimensional stream function
ζ	non-dimensional vorticity

Subscripts

e	inside of the sphere
s	surface of the sphere
∞	free stream

Superscript

*	dimensional quantity

attached separation bubble to a steady non-axisymmetric wake consisting of a shortened separation bubble with two trailing counter-rotating vortices occurs at approximately at $Re = 211$. Subsequently, Johnson and Patel [8], Tomboulides and Orszag [9] and Thomson et al. [10] found that the sphere wake experiences a transition to a steady asymmetric wake from a steady axisymmetric wake at a Reynolds number which lies in the range 210–220. As Reynolds number increases above 270, the flow is three-dimensional and time-dependent with periodic vortex shedding [11,12]. Bagchi et al. [12] studied the vortex shedding phenomena and its effect on heat transfer from a sphere. Yun et al. [13] studied the vortical structure behind a sphere at a subcritical Reynolds number. From those studies it may be noted that vortex shedding has a difference in characteristics from that of a circular cylinder. The vortex shedding from a circular cylinder occurs for Reynolds number above 40. In case of a circular cylinder, shedding of vortices occurs from either side of the cylinder where as, vortices are shed only from the top of the sphere.

Heat transfer from or to a body of spherical or near spherical shape is a problem of great practical importance. Acrivos and Taylor [14], Bernner [15], and Dennis et al. [16] investigated the heat and mass transfer from spherical and arbitrary shaped bodies at small Peclet numbers in forced convection dominated regime. In a recent article by Feng and Michaelides [17] and in the book by Michaelides [3] provided a detail account on recent advances on the analytical form of the hydrodynamic force and heat transfer from spherical particles in slow motion. The steady state natural convection over a sphere has been studied numerically by Jia and Gogos [18] for a wide range of Grashof numbers and who have also made a discussion on some of the previous work. Their results show that a steady state buoyancy plume with a mushroom-shaped front forms above the sphere whose length and thickness decrease with increasing Grashof number. Recently, Yang et al. [19] studied a similar natural convection problem for a wider range of Grashof number and Prandtl number.

The convection of heated drops and particles through a fluid medium induces a disturbance to the host fluid due to the buoyant force. The mixed convection about a point heat source is made by Riely and Darke [20] through a boundary-layer analysis. Subsequently, Riely and Tveitereid [21] made the linear stability analysis due to an axisymmetric buoyant plume above a point heat source in presence of a co-flowing vertical stream. Their analysis shows that the forced flow has a stabilizing effect. The local vorticity and the local potential energy in the plume are reduced by the introduction of the external stream. The sedimentation of solid particles in a hotter or colder fluid within a vertical channel is studied by Gan et al. [22]. They made a two-dimensional study with particles represented as circles. Due to the two-dimensional simulation, vortex shedding sets-in and wake becomes oscillatory at a much lower Reynolds number as compared to the case where the particles are considered to be spherical.

The combined convection due to a solid sphere has been studied by Nguyen et al. [23] for different values of Reynolds number and Grashof number. There the authors provided a discussion on some of the related studies based on either boundary layer solution or solution of two-dimensional Navier–Stokes equations. However, their study did not provide a detailed analysis on the wake characteristics and its influence on the heat transfer form the sphere for a wider range of flow parameters. The experimental studies