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On natural convection in enclosures filled with fluid-saturated porous media including viscous dissipation

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Abstract

Care needs to be taken when considering the viscous dissipation in the energy conservation formulation of the natural convection problem in fluid-saturated porous media. The unique energy formulation compatible with the First Law of Thermodynamics informs us that if the viscous dissipation term is taken into account, also the work of pressure forces term needs to be taken into account. In integral terms, the work of pressure forces must equal the energy dissipated by viscous effects, and the net energy generation in the overall domain must be zero. If only the (positive) viscous dissipation term is considered in the energy conservation equation, the domain behaves as a heat multiplier, with an heat output greater than the heat input. Only the energy formulation consistent with the First Law of Thermodynamics leads to the correct flow and temperature fields, as well as of the heat transfer parameters characterizing the involved porous device. Attention is given to the natural convection problem in a square enclosure filled with a fluid-saturated porous medium, using the Darcy Law to describe the fluid flow, but the main ideas and conclusions apply equally for any general natural or mixed convection heat transfer problem. It is also analyzed the validity of the Oberbeck–Boussinesq approximation when applied to natural convection problems in fluid-saturated porous media.

1. Introduction

There is an increasing interest in the study of natural convection in fluid-saturated porous media, as proved by the explosive growth in the literature on the subject, and also an increasing interest in the consideration of the viscous dissipation effects on the flow and temperature fields, as well as on the heat transfer performance of the involved devices. From an order of magnitude analysis it can be concluded that the viscous dissipation can be neglected in many situations of practical interest, both for domains filled with a clear fluid or for domains filled with fluid-saturated porous media. This is, however, a subject that attracts many research workers and, in particular, special attention is being devoted to the natural convection in enclosures filled with a fluid-saturated porous medium including the viscous dissipation effects. In this work, it is studied the natural convection in a square enclosure, but the main results and conclusions apply to any natural or mixed convection problem in fluid-saturated porous media. The corresponding problem, relative to a square enclosure filled with a clear fluid, has been studied recently by Costa [1], and the interest on this problem can be assessed by the references cited herein and also by the very recent works of Pons and Le-Queré [2–4].

Going on to the literature, one can find many recent works concerning the natural convection in fluid-saturated porous media, including viscous dissipation effects. Examples of works considering the Darcy Law to describe the fluid flow are these of Nakayama and Pop [5], Magyari and Keller [6], Rees et al. [7], Saeid and Pop [8] and Rees [9]. In the work of Al-Hadhrami et al. [10] it is considered the Brinkman extension of the Darcy Law, and a quadratic drag term on the momentum equation is considered in the works of Murthy and Singh [11], Murthy [12], Tashtoush [13] and Magyari et al. [14]. The book by Nield and Bejan

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Nomenclature

c_P	constant pressure specific heat
Da	Darcy number
E	total energy
Ec	Eckert number
g	gravitational acceleration
$\mathbf{g}$	gravitational acceleration vector
H	height
$\hat{h}$	specific enthalpy
k	thermal conductivity
K	permeability
m	mass
Nu	Nusselt number
p	pressure
Pr	Prandtl number
$\mathbf{q}$	heat flux vector
$\dot{Q}$	heat flow
Ra	Darcy-modified Rayleigh number
$\dot{S}$	entropy flow
$\dot{S}'''$	volumetric rate of entropy generation
t	time
T	temperature
u, v	Cartesian velocity components
$\hat{u}$	specific internal energy
V	volume
$\mathbf{v}$	surface velocity vector
$\mathbf{V}$	intrinsic velocity vector
$\dot{W}$	mechanical power
x, y	Cartesian co-ordinates

Greek symbols

a	thermal diffusivity
β	volumetric expansion coefficient
ΔT	temperature difference
ε	porosity
μ	dynamic viscosity
ν	kinematic viscosity
ρ	density
τ	temperature ratio
ψ	(conservative) streamfunction

Subscripts

C	cold (lower temperature) value
CD	conduction
D	viscous dissipation
f	fluid
gen	generation
H	hot (higher temperature) value
m	combined
0	reference value
s	solid
*	dimensionless

[15] gives a very good description about the relevance of the subject of heat transfer in porous media, and about the models used to take into account the different effects on the natural convection in fluid-saturated porous media. Nield [16] gives an explanation why the quadratic drag term on the momentum equation (which does not contain the viscosity in an explicit way) must be taken into account as a dissipation term. A study of the entropy generation associated with the natural convection heat transfer problem in an inclined square enclosure filled with a fluid-saturated porous medium was conducted by Baytas [17]. In this work, the viscous dissipation term is not taken into account in the energy conservation equation, but it is taken into account in the entropy generation equation.

In all the previously referred works, concerning natural convection in fluid-saturated porous media, with the exception of the book by Nield and Bejan [15], no reference is made to the work of pressure forces term when the viscous dissipation term is taken into account in the energy conservation equation. In fact, as it is shown in the present work, both terms need to be considered in order to have the unique energy conservation formulation that is consistent with the First Law of Thermodynamics.

If we consider an enclosure filled with a fluid-saturated porous medium, like the one presented in Fig. 1, where steady natural convection takes place, and the viscous dissipation term is considered in the energy conservation equation, a net heat generation takes place in the enclosure and heat leaving the enclosure is greater than that entering the enclosure. Such an enclosure behaves like a heat multiplier, which is inconsistent regarding the First Law of Thermodynamics. It must be noted that viscous dissipation is due to fluid motion, and that fluid motion is not forced

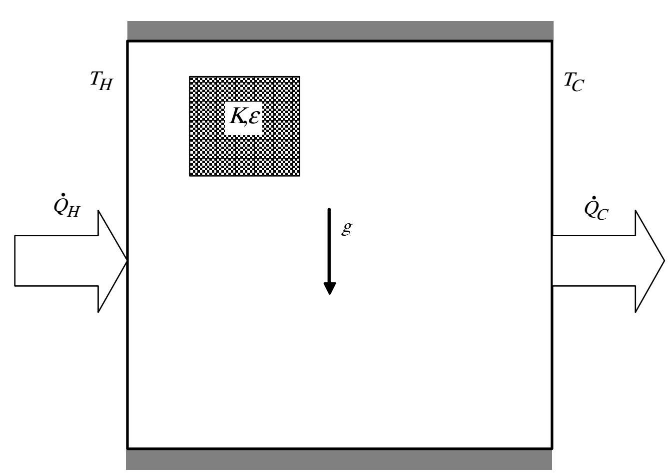

Fig. 1. The natural convection problem in a differentially heated square enclosure filled with a fluid-saturated porous medium.