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Pattern formation of drops in thermocapillary migration

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Abstract

The behavior of a drop cloud in thermocapillary motion in zero gravity is examined for both mono-dispersed and poly-dispersed cases. Numerical simulations of the thermocapillary motion of two- and three-dimensional fully deformable light drops are presented. The Navier–Stokes equations coupled with the energy conservation equation are solved by a front-tracking/finite-difference method. The material properties of the drop fluid and the ambient fluid are different, and the interfacial tension depends on the temperature. At moderate Reynolds (*Re*) and Marangoni (*Ma*) numbers, the results show that drops form layers nearly perpendicular to the temperature gradient.

Keywords: Thermocapillary migration; Zero gravity; Pattern formation; Front-tracking method

1. Introduction

It is well known that drops suspended in a fluid with a temperature field move in the direction of the temperature gradient due to thermocapillary forces. Interfacial tension generally decreases with increasing temperature and a non-uniform temperature field in the ambient fluid generates an interfacial tension gradient at the fluid interface that, in turn, induces shear stresses acting on the outer fluid by viscous forces, and thus inducing a motion of the drop in the direction of the temperature gradient. This phenomenon is known as the thermocapillary migration of drops and it can play an important role in material processing under the microgravity condition in the space as well as in many other scientific and engineering applications.

Following the pioneering work of Young et al. [14] who found an analytical expression for the terminal velocity of a single spherical drop in the creeping flow limit, the behavior of a single fluid particle in a temperature gradient has been extensively studied and been reasonably well understood. However, it is frequently necessary to deal with a large number of drops and their collective behavior may differ substantially from the thermocapillary migration of a single isolated drop. The thermocapillary motion of two drops and their interactions were first examined analytically by Anderson [1] in the limit of zero Reynolds and Marangoni numbers. Anderson [1] showed that the collective behavior of a droplet suspension is considerably different from that of a single isolated drop. This result was then confirmed by Keh and Chen [3] who studied axisymmetric thermocapillary migration of two spherical droplets in a creeping flow regime. Keh and Chen [4] also investigated the axisymmetric thermocapillary motion of a chain of spherical droplets in a quasi-steady state limit of conservation of energy and momentum using a combined analytical–numerical method. The interaction of dispersed spherical drops in thermocapillary motion was examined by Zhang and Davis [16] in creeping flow conditions with a trajectory method. Keh and Chen [5] studied the

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Nomenclature

a	initial drop radius (m)	We	Weber number ($\rho_o a U_r^2/\sigma_o$)
a_v	average drop radius (m)	α	thermal diffusivity ($k/\rho c_p$)
c_p	specific heat capacity (J/kg K)	γ	deformation
c_p^*	ratio of heat capacity (c_{p_i}/c_{p_o})	ϵ	deformation parameter
i	subscript denoting drop fluid	μ	dynamic viscosity (N s/m^2)
k	heat conduction coefficient (W/m K)	μ^*	viscosity ratio (μ_i/μ_o)
k^*	ratio of heat conduction coefficient (k_i/k_o)	ν	kinematic viscosity (m^2/s)
Ma	Marangoni number ($U_r a/\alpha_o$)	ρ	density (kg/m^3)
o	subscript denoting ambient fluid	ρ^*	density ratio (ρ_i/ρ_o)
Re	Reynolds number ($U_r a/\nu_o$)	σ	interfacial tension coefficient (N/m)
T_o	reference temperature (K)	σ_o	interfacial tension coefficient at T_o
t_r	reference time scale (a/U_r)	σ_T	proportionality coefficient ($-\mathrm{d}\sigma/\mathrm{d}T$)
U_r	reference velocity ($\sigma_T a\lvert\nabla T_\infty\rvert/\mu_o$)	∇T_∞	temperature gradient in undisturbed ambient fluid
U_{rv}	average reference velocity (m/s)		

interaction of many droplets in the limit of creeping thermocapillary motion and found that the terminal velocity of gas bubbles is independent of each other if they are all equal in size in the limiting case of zero Reynolds and Marangoni numbers. Nas and Tryggvason [9] studied the interaction of two droplets at moderate Reynolds and Marangoni numbers and showed that, in contrast with the results found in the creeping flow limit, the terminal velocity of droplets can be strongly affected by the presence of other droplets depending on the separation distance between them. The reader is referred to the review papers by Subramanian [10] and by Wozniak et al. [15] and to a recent book by Subramanian and Balasubramaniam [11] for a detailed discussion of analytical, numerical and experimental methods about the thermocapillary motion of drops in reduced gravity including a more complete list of literature on the subject.

The investigations of interactions of drops discussed above have mostly been limited to zero Reynolds and Marangoni numbers. In many engineering applications where thermocapillary forces are dominant, it is likely that many drops are present and heat and mass convections are important, i.e., Reynolds and Marangoni numbers are non-zero. It is therefore critical to understand the overall behavior of large drop systems with either mono-dispersed or poly-dispersed cases including the effect of non-zero Reynolds and Marangoni numbers. In the present work, numerical simulation of equal size (mono-dispersed) drops as well as unequal size (poly-dispersed) drops in two and three dimensions are presented for non-zero values of Reynolds and Marangoni numbers. It is found that the drops align themselves nearly perpendicular to the temperature gradient. This might be an important result since it suggests that the formation of drop layers may result in dislocations inside the solidified material produced in microgravity environment.

2. Formulation and numerical method

The governing equations are described in this section in the framework of the front-tracking method. In this method, the flow equations are written for the entire flow field and different phases are treated as a single fluid with variable material properties. A detailed description and numerical properties of the front-tracking/finite-difference method can be found in the review paper by Tryggvason et al. [12] where some results for the thermocapillary migration of drops are also presented. In addition, the formulated governing equations and the numerical solution method employed here for the computations of thermocapillary migration of drops are the same as described by Nas and Tryggvason [9]. Some validation results and an extensive computational study of the thermocapillary motion of a single drop and two drop interactions can also be found in Nas and Tryggvason [9].

2.1. Governing equations

As mentioned above, it is possible to write the Navier–Stokes equations as a single set of equations for the whole domain as long as the jumps in fluid properties are correctly accounted for and interfacial tension is included. The Navier–Stokes equations in conservative form are given by

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u}\mathbf{u}) = -\nabla p + \nabla \cdot \mu(\nabla \mathbf{u} + \nabla \mathbf{u}^{\mathrm{T}}) + \int \delta^{\beta}(\mathbf{x} - \mathbf{x}_{\mathrm{f}}) \frac{\partial}{\partial s}(\sigma \mathbf{t})\mathrm{d}s, \tag{1}$$

where the last term is the interfacial tension acting on the interface, included as a body force by representing it as a delta function. Here $\mathbf{u}$ is the velocity field, p is the pressure,