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Statistical modeling of particles relative motion in a turbulent gas flow

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Abstract

On the base of modern probability approach the theoretical model of turbulent relative motion of particles in the turbulent flow is developed. Closed equation for probability density function of coordinates and velocities of two particles in turbulent flow is obtained. The system of equations for balance of mass, averaged velocities and intensities of turbulent chaotic motion of particles with account of correlated motion of particles are deduced. The closed expressions for intensity of relative chaotic motion between particles are obtained on the base of probability density function of particles displacement with correlation effects. The correlation functions, intensity of relative turbulent motion and relative diffusion coefficients of particles are numerically investigated. The calculation results are compared with data of large eddy simulations. The results of calculation intensity of droplets relative motion in atmospheric conditions are presented.

1. Introduction

In gas flows the rate of particles or droplets coagulation depends on their relative velocity and collision frequencies. The relative velocity of particles is determined by the external forces, for example, mass forces as well as particles intensity of random motion in the turbulent flow. The paper is devoted to investigation the relative turbulent transport of particles with various sizes. The entrainment of particles in turbulence depends on their inertia. Small particles, whose dynamic relaxation time is much less than the integral time scale of turbulence are completely entrained in the turbulent motion of energy containing eddies. Without consideration the effect of particles inertia on the degree of entrainment in the small-scale turbulence, the averaged relative velocity between particles is determined by a gradient of a carrier phase velocity on a distance of order the sum of particles diameters [1,2]. In [3], within the framework of the model outlined in [1] the small-particles coagulation kernel was calculated with allowance for Brownian and relative turbulent diffusion. In [3] the effect of relative averaged velocity slips between particles due to gravity force was includes in efficiency of particles coagulation. In a gas flow small inertia particles have diameters lesser than Kolmogorov space micro scale. For such particles relative velocity due to gradient of fine grained turbulence on a distance of particles diameters is negligible. Trajectories of these particles are well correlated and chaotic relative velocity of small inertia particles equal to zero.

For inertial particles with dynamic relaxation time of order integral time macro scale of turbulence the intensity of their chaotic motion is determined by entrainment of particles into turbulent motion of energy containing eddies. These particles do not participate into small scale high frequency turbulence. In [4] it is assumed, that trajectories of inertial particles are not correlate. In [4] by analogy of kinetic theory of gaseous the energy of random motion of two particles was set as a sum of energy of chaotic motion of the particles. The degree of entrainment of particles into turbulent motion of large eddies was taken into account in [5]. In the [5] the approximate distribution

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Nomenclature

$D_{\alpha,ik}$ coefficient of turbulent diffusion of αth particles
$D_{\alpha\beta,ik}$ coefficient of turbulent relative diffusion between two particles
$D_{\circ}$ coefficient of turbulent diffusion of inertia less particles
d_p diameter of a particle
f_{α}, $f_{\alpha|\beta}$ unconditional and conditional response functions
G_{α}, $G_{\alpha\beta}$ probability density functions of particles transfer
$\mathbf{g}$ gravity acceleration
L_E Euler integral space scale
N_{α} distribution function of one type particles in space
$N_{\alpha\beta}$ distribution function of particles of two types in space
q_{α}, $q_{\alpha|\beta}$ unconditional and conditional response functions
Re_{λ} Reynolds number calculated on Taylor microscale
T_E Euler integral temporary scale
T_L Lagrange temporary scale
T_{α} temporary scale of gas velocity fluctuations along αth particle path
$\mathbf{U}$ actual velocity of fluid phase
$\mathbf{u}$ velocity fluctuations of fluid phase
$\mathbf{V}_{\alpha}^{(p)}$ actual velocity of the αth particle
$\mathbf{V}_{\alpha}$ Euler velocity of αth particle
$\mathbf{v}_{\alpha}^{(p)}$ velocity fluctuations of αth particle
$\mathbf{X}_{\alpha}^{(p)}$ Lagrange position of αth particle
$\mathbf{x}_{\alpha}$ Euler position of αth particle
$\mathbf{Y}_{\alpha\beta}$, $\mathbf{y}_{\alpha\beta}$ relative distances between two particles
$\mathbf{W}_{\alpha}$ averaged velocity of αth particle due to mass force
$\mathbf{W}_{\alpha\beta}$ averaged relative velocity between two particles
$\mathbf{w}_{\alpha\beta}$ turbulent relative velocity between two particles

Greek symbols
γ_{α} nondimensional relative velocity of αth particle
Δ total dispersion of particles turbulent transfer
$\delta(\mathbf{x})$ three-dimensional Dirac delta-function
ε turbulent dissipation rate
Λ dispersion of particles turbulent transfer due to inertia
λ dispersion of particles transfer with energy containing eddies
μ ratio between Lagrange and Euler temporary scales
$\rho_{\alpha\beta}$ coefficient of two particles velocity correlation
σ_{α}, $\sigma_{\alpha\beta}$ second moments of particles velocity fluctuations
τ_{α} dynamic relaxation time of αth particle
$\Phi_{\alpha\beta}$ indicator function for two particles
$\varphi_{\alpha\beta}$ probability density function of two particles velocity distribution
χ structural parameter of turbulent flow
Ψ_E Euler correlation function
$\Psi_{\alpha}^{(p)}$ unconditional gas velocity correlation function
$\Psi_{\alpha|\beta}^{(p)}$ conditional gas velocity correlation function
Ω_{α} parameter of inertia of αth particle

Subscripts
α,β particles αth and βth types
$\langle\ \rangle$ denotes result of averaging over an ensemble of turbulent realizations

of turbulent energy of carrying phase was involved for calculation the intensity of random motion of particles with different sizes. But in [5] was assumed, that trajectories of inertial particles with equal sizes are completely correlated. So, in a turbulent gas flow inertial particles with equal diameters are not colliding with each other.

The large eddy simulations (LES) and direct numerical simulations was used in [6,7] for investigation the particles collisions in the homogeneous turbulent motion. In these works the role of trajectory correlations of inertial particles are brightly illustrated. In has been established that inertia less particles move in very correlated manner. With increasing particles inertia correlation between particles trajectories destroys and the relative turbulent velocity increases. For very inertial particles, whose dynamic relaxation times are mach larger then integral time scales of turbulence, the intensity of all turbulent motion of dispersed phase fall down. In [6] theoretical model for calculation the relative motion of particles with equal sizes was suggested. For calculation intensity of particles random motion was used Boltzmann hypothesis from kinetic theory of gaseous. The approach [6] is valid for particles with equal sizes and do not take into account effect of reduction of correlation between particles with increasing relative distance. In the models [6] the relative turbulent diffusion of particles is not considered. But the contribution of relative turbulent diffusion of particles in relative turbulent motion of particles is very important.

The perspective modern approach for investigation particles relative chaotic motion based on probability density function (PDF) for particles coordinates and velocities was suggested in [8]. The closing PDF equation has been achieved due to assumption that relative displacement of particles is a result only of relative random velocity between particles. The model [8] is valid for description relative chaotic motion of particles with equal sizes. We can note that in the specific case of particles with equal