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Simulation of turbulent natural convection in a porous cylindrical annulus using a macroscopic two-equation model

Edimilson J. Braga, Marcelo J.S. de Lemos *

Departamento de Energia – IEME, Instituto Tecnológico de Aeronáutica – ITA, 12228-900 São José dos Campos, SP, Brazil

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Abstract

This work presents numerical computations for laminar and turbulent natural convection within a horizontal cylindrical annulus filled with a fluid saturated porous medium. Computations covered the range $25 < Ra_m < 500$ and $3.2 \times 10^{-4} > Da > 3.2 \times 10^{-6}$ and made use of the finite volume method. The inner and outer walls are maintained at constant but different temperatures. The macroscopic k–ε turbulence model with wall function is used to handle turbulent flows in porous media. First, the turbulence model is switched off and the laminar branch of the solution is found when increasing the Rayleigh number, Ra_m. Subsequently, the turbulence model is included and calculations start at high Ra_m, merging to the laminar branch for a reducing Ra_m. This convergence of results as Ra_m decreases can be seen as an estimate of the so-called laminarization phenomenon. Here, a critical Rayleigh number was not identified and results indicated that when the porosity, Prandtl number, conductivity ratio between the fluid and the solid matrix and Ra_m are kept fixed, the lower the Darcy number, the higher is the difference of the average Nusselt number given by the laminar and turbulent models.

Keywords: Turbulence modeling; Porous media; Natural convection; Cylindrical annuli

1. Introduction

The analyses of natural convection in a horizontal cylindrical annuli filled by a porous material has been subject of a number of studies in recent years. Thermal insulators, cryogenics, thermal storage systems, electronic cooling, inert gas insulation of high-voltage electric cables and the determination of the requirements for aircraft cabin insulation are some examples of applications.

The first basic study on natural convection in clear cylindrical annuli was carried out by [1] and extended by [2]. A very extensive analysis has been made on the concentric annuli by [3]. They have conducted both numerical simulation using the finite elements technique and experimental studies using Mach-Zehnder interferometer. Application of other type of finite differences method with ADI numerical solution has also reported by [4] in solving the laminar horizontal concentric annuli problem formulated in cylindrical polar coordinates. Small eccentric annuli has performed in the work of [5], using an expansion in terms of the double series of eccentricity and Rayleigh number for small values of Ra. The work of [6] extended the knowledge on the natural convection heat transfer in the horizontal cylindrical annuli and the numerical analysis has been made using finite difference method in a bipolar coordinate system based on successive over-relaxation iteration.

Flow analysis on clear concentric annulus has been extended to turbulent natural convection using a two-equation turbulence model in the work of [7]. Experimental works yielded new information about the field variable, primarily on the distribution of the mean temperature in the fluid and of Nusselt number around the cylinders circumference in the work of [8]. The turbulence appears in the upper region of the annulus in the plume above the heated inner cylinder. On the other hand, the stable stratification below

* Corresponding author. Tel.: +55 12 3947 5860; fax: +55 12 3947 5842.
E-mail address: delemos@mec.ita.br (M.J.S. de Lemos).

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Nomenclature

c_F	Forchheimer coefficient
c's	non-dimensional turbulence model constants
c_p	specific heat
d	pore diameter
$\mathbf{D}$	$\mathbf{D} = [\nabla\mathbf{u} + (\nabla\mathbf{u})^T]/2$, deformation rate tensor
Da	Darcy number, $Da = \frac{K}{r_i^2}$
D_p	particle diameter
$\mathbf{g}$	gravity acceleration vector
G^i	generation rate of $\langle k\rangle^i$ due to the action of the porous matrix
G^i_β	generation rate of $\langle k\rangle^i$ due to the buoyant effects
h	heat transfer coefficient
$\mathbf{I}$	unit tensor
K	$K = \frac{D_p^2\phi^3}{144(1-\phi)^2}$, permeability
k	$k = \overline{\mathbf{u}'\cdot\mathbf{u}'}/2$, turbulent kinetic energy per mass unit
k_f	fluid thermal conductivity
k_s	solid thermal conductivity
$\mathbf{K}_{disp}$	conductivity tensor due to the dispersion
$\mathbf{K}_{disp,t}$	conductivity tensor due to the turbulent dispersion
$\mathbf{K}_t$	conductivity tensor due to the turbulent heat flux
$\mathbf{K}_{tor}$	conductivity tensor due to the tortuosity
Nu	Nusselt number
Pe	Peclet number
Pe_D	modified Peclet number, $Pe_D = Pe(1-\phi)^{1/2}$
P^i	production rate of $\langle k\rangle^i$ due to gradients of $\bar{\mathbf{u}}_D$
Pr	ν_f/α_f, Prandtl number
r	radius
R	r_o/r_i
Ra_f	$Ra_f = \frac{g\beta r_i^3\Delta T}{\nu_f\alpha_f}$, fluid Rayleigh number
Ra_m	$Ra_m = Ra_f\cdot Da = \frac{g\beta_\phi r_i\Delta TK}{\nu_f\alpha_{eff}}$, Darcy–Rayleigh number
Ra_{cr}	critical Rayleigh number
Re_p	$Re_p = \frac{\lvert\mathbf{u}_D\rvert d}{\nu_f}$, Reynolds number based on the pore diameter
T	temperature
$\mathbf{u}$	microscopic velocity
$\mathbf{u}_D$	Darcy or superficial velocity (volume average of $\mathbf{u}$)

Greek symbols

α	thermal diffusivity
β	thermal expansion coefficient
ΔV	representative elementary volume
ΔV_f	fluid volume inside ΔV
ε	$\varepsilon = \mu\overline{\nabla\mathbf{u}' : (\nabla\mathbf{u}')^T}/\rho$, dissipation rate of k
μ	dynamic viscosity
μ_t	microscopic turbulent viscosity
μ_{t_ϕ}	macroscopic turbulent viscosity
ν	kinematic viscosity
ρ	density
σ's	non-dimensional constants
ϕ	$\phi = \Delta V_f/\Delta V$, porosity
θ	angle

Special characters

φ	general scalar variable
$\bar{\varphi}$	time average
φ'	time fluctuation
$\langle\varphi\rangle^i$	intrinsic average
$\langle\varphi\rangle^v$	volume average
${}^i\varphi$	spatial deviation
$\lvert\varphi\rvert$	absolute value (Abs)
$\boldsymbol{\varphi}$	general vector variable
φ_{eff}	effective value, $\varphi_{eff} = \phi\varphi_f + (1-\phi)\varphi_s$
$\varphi_{s,f}$	solid/fluid
$\varphi_{i,o}$	inner/outer
$\varphi_{H,C}$	hot/cold
φ_ϕ	macroscopic value
$(\ )^T$	transpose

the heated inner cylinder suppresses any movement and turbulence that might be convected downward. *Direct numerical simulation* (DNS) was performed by [9]. However, because of the extreme requirements for computing resources, these computations are limited to low Ra_f numbers. The work of [10] use a *Large Eddy Simulation* (LES) for a case with a high Ra_f number indicated that this method may serve in the future as an alternative to the (DNS), but there are many critics about the use of such model in 2-D cases due the 3-D characteristic of the turbulence. The work of [11], reports on the modeling and computational study of the natural convection in concentric and eccentric annuli by means of several variants of the Algebraic Stress Models (ASM), which is based on the expression for turbulent heat flux $\overline{\theta u_i}$ obtained by truncation of the second-moment transport equation for this correlation. Various levels of closure included the Low-*Re* number form of the k–ε model, but also a version in which the differential transport equations are solved for the temperature variance $\overline{\theta^2}$ and its decay rate ε_θ. Ascending flows of liquid metal [12] and anisotropic effects in flows over rod-bundles, [13] have also been investigated via ASMs.

The natural convection in cylindrical annular geometry filled with porous material also have been studied by distinct numerical approaches, such as the finite-difference method reported by [14 and 15]. Finite element method is found in the work of [16] and the Galerkin spectral method in the work of [17–20]. The work of [21] have shown that the Fourier–Chebyshev method gives better accuracy than