Available online at www.sciencedirect.com

SCIENCE DIRECT®

International Journal of Heat and Mass Transfer 49 (2006) 4552–4557

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Constructal design of Y-shaped assembly of fins

Giulio Lorenzini [a,*], Luiz Alberto Oliveira Rocha [b]

[a] *Department of Agricultural Economics and Engineering, Alma Mater Studiorum-University of Bologna, viale Giuseppe Fanin no. 50, 40127 Bologna, Italy*

[b] *Departamento de Física, Fundação Universidade Federal do Rio Grande, Cx.P. 474, Rio Grande, RS 96201-900, Brazil*

Received 10 February 2006
Available online 25 July 2006

Abstract

This work relies on constructal design to perform the geometric optimization of the Y-shaped assembly of fins. It is shown numerically that the global thermal resistance of the Y-shaped assembly of fins can be minimized by geometric optimization subject to total volume and fin material constraints. A triple optimization showed the emergence of an optimal architecture that minimizes the global thermal resistance: an optimal external shape for the assembly, an internal optimal ratio of plate-fin thicknesses and an optimal angle between the tributary branches and the horizontal. Parametric study was performed to show the behavior of the minimized global thermal resistance. The results also show that the optimized Y-shaped structure performs better than the optimized T-shaped one.

Keywords: Constructal theory; Enhanced heat transfer; Fins; Optimization

1. Introduction

Constructal design has been applied to a large variety of engineering problems, e.g., [1–4], to optimize shape and structure [5]. On the other hand, the augmentation of heat transfer has been pursued for a long time in the engineering field. Individual fins and assemblies of fins have been studied exhaustively and the results can be found in some reviews [6,7]. Recently, constructal design has been applied successfully to the geometrical optimization of fins. Bonjour et al. [8] documents the fundamental relation between the maximization of global performance and the malleable (morphing) architecture in the coaxial two-stream heat exchanger. Configurations with radial and branched fins were optimized. Vargas et al. [9] conducted a combined numerical and experimental study to maximize heat transfer by optimizing a finned circular and elliptic tubes heat exchangers.

Constructal design has also been used in the study of cavities, i.e., inverted or negative fins. Biserni et al. [10] optimized C- and T-shaped cavities while Rocha et al. [11] optimized the trapezoidal external shape of C-shaped cavities. Both of the works minimized the global thermal resistance while the total volume and the cavity volume were kept as constrains. Bejan and Almogbel [12] optimized several types of assembly of fins that have been recognized in practice including the T-shaped assembly of fins. The objective of the T-shaped constructal optimization was to maximize the global thermal conductance of the assembly subject to total volume and fin-material constraints. The two degrees of freedom of the T-shaped structure were the external shape and the internal ratio of plate-fin thickness for the assembly.

This work relies on the constructal design to optimize the complete geometry of the Y-shaped assembly of fins, i.e., the T-shaped structure version with an additional degree of freedom: the angle between a tributary branch and the horizontal. The objective is to minimize the global thermal resistance subject to the total volume and fin-material constraints.

* Corresponding author. Tel.: +39 051 2096186; fax: +39 051 2096178.
E-mail addresses: giulio.lorenzini@unibo.it (G. Lorenzini), dfsrocha@furg.br (L.A. Oliveira Rocha).

0017-9310/$ - see front matter
doi:10.1016/j.ijheatmasstransfer.2006.05.019

Nomenclature

a	dimensionless parameter, Eq. (9)
A	area [m^2]
h	heat transfer coefficient [W m^{-2} K^{-1}]
k	fin thermal conductivity [W m^{-1} K^{-1}]
L	length [m]
q	heat current [W]
t	thickness [m]
T	temperature [K]
V	volume [m^3]
W	width [m]

Greek symbols

α	angle between the tributary branches and the horizontal
θ	dimensionless temperature, Eq. (5)
ϕ	volume fraction of fin material

Subscripts

f	fin material
m	single optimization
mm	double optimization
mmm	triple optimization

Superscript

$(\tilde{\ })$	dimensionless variables, Eqs. (6), (7), (10), (11), (14) and (15)

2. Mathematical model

Consider the Y-shaped assembly of fins sketched in Fig. 1. Two elemental fins of thickness t_0 and length L_0 serve as tributaries to a stem of thickness t_1 and length L_1. The elemental fin of thickness t_0 forms an angle α with a horizontal line. The configuration is two-dimensional, with the third dimension (W) sufficiently long in comparison with L_0 and L_1. The heat transfer coefficient h is uniform over all the exposed surfaces. The heat current through the root section (q_1) and the temperature of the fluid (T_∞) are known. The maximum temperature ($T_{1,\max}$) occurs at the root section ($y = 0$) and varies with the geometry.

The objective of the analysis is to determine the optimal geometry ($L_1/L_0, t_1/t_0, \alpha$) that is characterized by the minimum global thermal resistance $(T_{1,\max} - T_\infty)/q_1$. According to constructal design [5], this optimization is subjected to two constraints, namely, the total volume (i.e., frontal area) constraint,

$$A = (L_1 + L_0 \sin\alpha + t_0 \cos\alpha)(2L_0 \cos\alpha + t_1) \quad (1)$$

and the fin-material volume constraint,

$$A_f = L_1 t_1 + 2L_0 t_0 + t_1 t_0 \cos\alpha - t_0^2 \sin\alpha \cos\alpha \quad (2)$$

The latter can be expressed as the fin volume fraction

$$\phi = A_f/A \quad (3)$$

The analysis that delivers the global thermal resistance as a function of the assembly geometry consists of solving numerically the heat conduction equation along the Y-shaped assembly of fins where the fins are considered isotropic with constant thermal conductivity k

$$\frac{\partial^2\theta}{\partial\tilde{x}^2} + \frac{\partial^2\theta}{\partial\tilde{y}^2} = 0 \quad (4)$$

where the dimensionless variables are

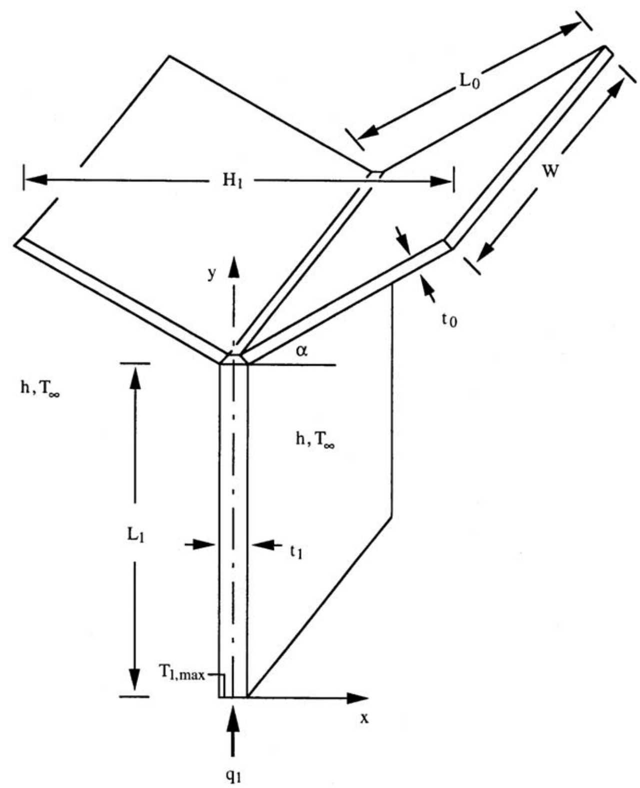

Fig. 1. Y-shaped assembly of fins analyzed.

$$\theta = \frac{T - T_\infty}{q_1/kW} \quad (5)$$

and

$$\tilde{x}, \tilde{y}, \tilde{t}_0, \widetilde{L}_0, \tilde{t}_1, \widetilde{L}_1 = \frac{x, y, t_0, L_0, t_1, L_1}{A^{1/2}} \quad (6)$$

The boundary conditions are given by

$$-\frac{\partial\theta}{\partial\tilde{y}} = \frac{1}{\tilde{t}_1} \quad \text{at } \tilde{y} = 0 \quad (7)$$