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Thermal lattice-BGK model based on large-eddy simulation of turbulent natural convection due to internal heat generation ☆

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Abstract

To simulate turbulent convection at high Rayleigh number (*Ra*), we propose a new thermal lattice-BGK (LBGK) model based on large eddy simulation (LES). Two-dimensional numerical simulations of natural convection with internal heat generation in a square cavity were performed at *Ra* from 10^6 to 10^{13} with Prandtl numbers (*Pr*) at 0.25 and 0.60. Simulation results indicate that our model is fit to simulate high *Ra* flow for its better numerical stability. At $Ra = 10^{13}$, a global turbulent has occurred. With a further increase in *Ra*, the flow will arrive in a fully turbulence regime. The Nusselt–Rayleigh relationship is also discussed.

Keywords: Thermal LBGK model; LES; Turbulent convection; High *Ra*

1. Introduction

Turbulent thermal convection is ubiquitous [1] in nature and technology, and serves important and diverse purposes. Natural convection (NC) flows due to internal heat generations has been lately receiving increase attention because of its relevance to nuclear safety issues. While the Rayleigh–Bénard convection (RBC) has been extensively studied, the literature on NC driven by internal heat is confined to fewer studies (see Ref. [2], for a list of references).

Turbulent convection sets in at high *Ra* [1,3–5]. However, it is still a challenge for both experiments and numerical simulations to capture the turbulent flow motion at high *Ra*. In fact, the highest *Ra* attainable in an apparatus of a given size is usually quite limited for a fluid such as water. Because of the unknown properties of the core melt at high temperatures, the researchers were unable to reproduce adequate accident conditions. Moreover, it is not a simple task to measure *Pr* dependence in convection turbulence by experiments. Therefore, numerical simulations are required to predict the turbulent flows especially at very high *Ra*.

For the smallest scales in the turbulent flows at high *Ra* are of the same magnitude with the Kolmogorov microscales, flows turbulence modeling becomes necessary. A viable alternative to the direct numerical simulation (DNS) is the method of LES [3,6], a time-honored method in engineering fluid mechanics and meteorology, which may be the only way to simulate the time-dependent physics in its full complexity while keeping a reasonable accuracy at the largest scales. Horvat et al. [2,7] simulated the NC flow with internal heat generation in a square cavity for a wide range of *Ra* and *Pr*: *Ra* 10^6–10^{13} and *Pr* 0.25–8 by using LES.

Since the NC flows at high *Ra* have complex behavior, efficient methods are still needed for further studies, especially for 3D problems. The LBGK method is a candidate for such methods [8,9]. For non-isothermal flows, several

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Nomenclature

c	particle speed
C	Smagorinsky constant
D	thermal diffusivity
D_t	turbulent thermal diffusivity
$\vec{e}_i$	discrete velocity
f_i, T_i	force term in Eqs. (2) and (5)
g	gravity
g_i, θ_i	distribution function for velocity and temperature field
g_i^{eq}, θ_i^{eq}	equilibrium distribution function for velocity and temperature field
L	length of simulation domain
I	volumetric heat generation
Nu	Nusselt number
Nu_{up}	Nusselt number for upper surface
p	pressure
Pr	Prandtl number ($=\nu/D$)
Pr_t	turbulent Prandtl number
Ra	Rayleigh number ($=g\beta IL^3/\nu D$)
S_{ij}	strain rate tensor
T	simulation time interval
$\vec{u}$	fluid velocity vector
$\vec{x}$	phase space

Greek symbols

β	temperature dilatation
ϵ	Knudsen number
θ	dimensionless temperature
ν	kinematic viscosity
ν_t	turbulent eddy viscosity
τ	relaxation time
$\tau_{\vec{u}}, \tau_{\vec{\theta}}$	relaxation time for velocity and temperature field
$\omega = \tau^{-1}$	relaxation parameter
ω_i	the weights for equilibrium distribution function
Δ	filter width
Δt	time step
Δx, Δy	grid spacing in x- and y-directions

Subscripts and superscripts

α, β	spatial index
i	discrete velocity direction
total	total index
0	initial index
$-$	filter operator

temperature LBGK (TLBGK) models have been developed [10–12]. However, most of these models suffer from complicated evolution equations [12] or severe numerical instability [13]. Recently, Guo et al. [14] propose a new TLBGK model with a robust boundary scheme. Although the model holds if viscous heating effects and compression work are negligible [15] and the boundary conditions simulate only imposed temperature [16], it has proved to have good stability as well as simplicity. Using the model, Shi [17] had succeeded in simulating the NC flows due to internal heat generation in a square cavity at Ra 10^6–10^{12} and Pr 0.25 and 0.60. However, the LBGK method is still viewed as a DNS so that it is limited to resolve relatively low Ra flows. For example, when $Ra = 10^{12}$ and $Pr = 0.25$, the simulation with the TLBGK model results in numerical instability. To extend the LBGK method to turbulent flows, it is natural to incorporate the existing turbulent models into the framework of the LBGK method. There are some paradigms which succeed in combining LES with the LBGK model to simulate isotherm flows at high Re [18–20]. Inspired by this idea, we propose a new TLBGK model based on LES which directly introduce the Smagorinsky eddy viscosity [6] to the TLBGK model. The approach will be presented in the second part of the paper.

To understand heat transfer at high Ra, the relationship between the Nusselt number (Nu) and Ra has drawn much attention as a scaling law [21–23]. However, the issue of the existence of an asymptotic regime that is supposed to occur at very high Ra remains an open one. Theory [21] predicted that a full turbulent regime arises in such a state $Nu = Ra^{\beta}$ with $\beta = 1/2$. But large amounts of convection experiments [4,22] revealed that some other exponents β such as 2/7 or 1/3 existed. Moreover, a recent theoretical study [23] has suggested that the $Nu(Ra)$ relation should not follow a strict power-law. As to the NC of internally heated fluids, a considerable amount of experimental and analysis effort was focused on determining the averaged Nu numbers on the cooled surface in many geometries such as fluid layer, rectangular, semicircular and elliptical cavities. Although the geometries were different, the relationship $Nu_{up}(Ra)$ for the upper surfaces are quite similar [24]. Hence, the question arises of whether or not a simple power-law relationship between Nu and Ra exists in turbulent natural convection due to internal heat generation. Since the fluid used in the above mentioned literatures were mostly water ($Pr = 2.5$–7) and Freon ($Pr = 8$–11), to answer the question, large numbers of numerical simulations at a large range Ra with relative low Pr are needed to be performed.

In the paper, we simulate natural convection flows due to internal heat generation in a cavity for Ra up to 10^{13} with Pr at 0.25 and 0.60 using the TLBGK model based on LES. Simulation results with isotherms and the time-boundary-averaged Nu vs. Ra figures are presented in the third part of the paper. Meanwhile, the relationship $Nu(Ra)$ is discussed. In the last part, conclusions are drawn based on these simulation results.