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Technical Note

Integral transform solution for natural convection in three-dimensional porous cavities: Aspect ratio effects

H. Luz Neto [a], J.N.N. Quaresma [b], R.M. Cotta [c,*]

[a] *National Institute of Technology – INT/MCT Ministério de Ciência e Tecnologia, Rio de Janeiro, RJ, Brazil*
[b] *Chemical and Food Engineering Department – DEQAL/UFPA, Universidade Federal do Pará Belém, PA, Brazil*
[c] *Mechanical Engineering Department – POLI/COPPE/UFRJ, Cidade Universitária, Federal University of Rio de Janeiro, Cx. Postal 68503, 21945-970 Rio de Janeiro, RJ, Brazil*

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Abstract

Three-dimensional natural convection in box-like cavities filled with a porous material is revisited, by considering a transient formulation for the energy balance and a quasi-steady formulation for the flow problem. The Generalized Integral Transform Technique (GITT) is employed in the hybrid numerical-analytical solution of the Darcy law based model for vertical cavities (insulated vertical walls with differentially prescribed horizontal wall temperatures), employing the vorticity-vector potential formulation. Comparisons with previously reported numerical solutions are performed and the transition between conductive and convective states is illustrated, centering on the aspect ratio influence on the flow and heat transfer phenomena. A set of reference results for the steady-state behavior under different aspect ratio is provided for covalidation purposes.

Keywords: Natural convection; Transition; Porous media; Rectangular cavities; Integral transforms

1. Introduction

Due to the large number of applications involving buoyancy induced flows in saturated porous media in different processes, such as those in the chemical, mechanical, environmental and geological fields (e.g., heat, mass and fluid flow in fixed bed chemical reactors, filtering processes, geothermal systems and petroleum reservoirs, to name a few), the prediction of natural convection in porous media filled cavities has deserved a broad scope of publications and reviews along the last few decades [1,2]. Following the same trend as in all branches of the physical sciences, a considerable amount of computer simulation work has been devoted to this fundamental problem in thermal sciences, aimed at producing benchmark results for the validation of general purpose computer software. Such refined simulation tasks have generally been achieved with the aid of conventional numerical techniques, and most frequently for mathematical formulations concerning two-dimensional geometries under different flow models. For three-dimensional formulations, due to the sometimes prohibitive computational effort associated with discretization processes, the literature is less abundant [3–13]. The available works for this situation in general adopt the Darcy flow model, together with the assumptions of constant and isotropic physical properties and linear variation with temperature of the buoyancy term (Boussinesq approximation). In addition, the cubic geometry is the most frequently one considered, and the situation of a vertical enclosure (a heated base and thermally insulated vertical walls) is commonly employed as the test case for the covalidation of solution methodologies.

Numerical results from all such different research efforts are far from coincident, seldom available in tabular form, and quite rarely for the full transient situations. Quite

* Corresponding author. Tel.: +55 21 2562 8567; fax: +55 21 2562 8383.
E-mail address: cotta@mecanica.coppe.ufrj.br (R.M. Cotta).

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Nomenclature

a, b	cavity width and depth, respectively
c_p	specific heat
d	cavity height
Da	Darcy number
g	gravity acceleration, $\mathbf{g} = (0, 0, -g)$
F_s	cavity shape factor
h_m	average heat transfer coefficient
K	permeability of solid matrix
k_m	thermal conductivity of porous medium, $k_m = \Phi k_f + (1-\Phi)k_s$
k_s	thermal conductivity of solid matrix
Mx, My	aspect ratio in x and y directions, respectively
Nu	Nusselt number, $Nu = h_m d/k_m$
$\overline{Nu}$, Nu_G	overall Nusselt number
Nu_C	characteristic overall Nusselt number
NT	truncation order of the eigenfunction expansion solution (number of terms)
Ra	modified Rayleigh number
t	time
T	temperature
V_C	cavity characteristic volume
(x^*, y^*, z^*)	space coordinates
(x, y, z)	dimensionless coordinates

Greek symbols

α_m	solid matrix thermal diffusivity – $\alpha_m = k_m/(\rho c_p)_{\text{fluid}}$
β	fluid thermal expansion coefficient
$\theta\ (x,y,z,\tau)$	dimensionless temperature
$\overline{\overline{\theta}}_{H_{imq}}(\tau)$	transformed temperature field
μ	fluid absolute viscosity
ν	fluid kinematic viscosity – $\nu = \mu/\rho$
ρ	fluid density
ρ_s	density of solid matrix
τ	dimensionless time
ψ	vector potential – $\psi\ (\psi_x, \psi_y, \psi_z)$
$\overline{\overline{\psi}}_{x_{imq}}$	transformed vector potential component in the x direction
$\overline{\overline{\psi}}_{y_{imq}}$	transformed vector potential component in the y direction

recently [14], the Generalized Integral Transform Technique (GITT) [15–17] has been employed in the development of a hybrid numerical–analytical solution for natural convection in the cubic cavity situation, allowing for a thorough comparison of previously published numerical results for the steady state Nusselt number. This hybrid approach has been demonstrated to be an efficient tool in the production of benchmark results in nonlinear diffusion and convection–diffusion problems and has been progressively advanced towards the automatic error-controlled solution of such partial differential problems. At this point, it is worth mentioning some illustrative contributions on this method for the specific class of problems of interest here, namely, the solution of natural convection problems in cavities under steady and transient regimen, for both porous media or just fluid filled two-dimensional enclosures [18–23]. The streamfunction-only formulation was preferred in all such contributions on natural convection because of the inherent advantages in its combined use with this hybrid approach, as more closely discussed in [15]. Later on, this hybrid solution scheme was advanced to handle the three-dimensional Navier–Stokes equations [24] based on the vector–scalar potentials formulation [25–27], with similar computational advantages with respect to the two-dimensional case. Since the pioneering work of Aziz and Hellums [25], the vorticity-vector potential approach has been receiving increasing attention, when it was shown that this formulation could lead to more stable and fast simulations of three-dimensional flows. This formulation was itself originally applied to three-dimensional natural convection in porous media [3].

The present contribution, following the efforts initiated in previous works that analyzed three-dimensional flows via integral transforms [14,24], is aimed at advancing this computational tool towards the accurate solution of natural convection within porous media filled rectangular three-dimensional cavities of arbitrary aspect ratio. We assume the Darcy flow model with the governing equations expressed in terms of the vorticity-vector potential formulation, considering a transient formulation for the energy balance and a quasi-steady formulation for the flow problem. The computer code was modified to produce reliable numerical results for the temperature field and Nusselt numbers for cavities of arbitrary aspect ratio, when reordering procedures and other computational aspects were emphasized in the improvement of the convergence behavior of the GITT approach, thus extending the algorithm constructed in [14]. The transition between conductive and convective states due to the variation of the cavity geometric parameters is also more closely examined.

2. Analysis

Three-dimensional natural convection in an impermeable box-like cavity filled with a porous material and saturated with a Newtonian fluid is considered. The flow is buoyancy induced by heat exchange between the fluid-porous media and the top and bottom walls.

The configuration considered (Fig. 1) is known as the vertical cavity problem, in which boundary conditions of first kind, T_0 and $T_0 + \Delta T$, respectively, are imposed at the top and at the bottom walls, whilst the vertical walls