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Heat transfer in the thermal entrance region for flow through rectangular porous passages

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Abstract

The study of heat transfer in rectangular passages with prescribed wall heat flux is of practical interest. These passages could be open or filled with saturated porous materials. A solution that uses the Green's function can accommodate the inclusion of heat flux over the entire surface area or over isolated sections of the boundary. Also, this solution permits the inclusion of frictional heating. Two different boundary conditions are considered: constant wall temperature and constant wall heat flux. The computed heat transfer coefficients show that the thermally fully developed condition may not be attainable in practical applications for very narrow passages with prescribed wall heat flux.

Keywords: Duct flow; Porous media; Forced convection; Weighted residuals; Rectangular ducts

1. Introduction

The placement of porous materials in passages can enhance the transfer of heat to a flowing fluid. Porous passages with rectangular cross-sections are useful devices for cooling of engineering systems. There has been a current interest in utilization of porous passages for electronic cooling applications; see e.g. [1]. Other applications are referenced in the review by Lage and Narasimhan [2]. A current general survey is contained in Nield and Bejan [3]. The particular topic of thermally developing forced convection in porous media is surveyed by Nield and Kuznetsov [4]. Recent papers involving porous-media forced convection in ducts of various shapes include those by Haji-Sheikh and Vafai [5] and Hooman and coworkers [6–8].

The computation of heat transfer rate in rectangular passages is the subject of this study. The temperature field in these passages may have different boundary conditions depending on the thermal conductivity of their impermeable enclosures. In this study, consideration is given to two different limiting boundary conditions that often appear in the literature: Constant uniform wall temperature and locally constant uniform wall heat flux. The first condition is appropriate when the thermal conductivity of the enclosing walls is sufficiently high. The prescribed local wall heat flux is the next limiting condition and it emerges when the uniformly heated walls of a passage are thin with relatively low conductivity. These two cases exhibit distinctly different and interesting features, especially in the thermally developing region. The analysis reveals that the coalescence of the thermal boundary layers from the opposite walls strongly depends on the distance between these walls if they are uniformly heated at a constant rate. For narrow rectangular passages, this phenomenon increases the length of the thermally developing region and makes the thermally fully developed condition unattainable in practical applications.

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Nomenclature

A	area (m^2)
$\mathbf{A}$	matrix
a	duct dimension, see Fig. 1
a_{ij}	elements of matrix $\mathbf{A}$
$\mathbf{B}$	matrix with elements b_{ij}
B_m	coefficients
b	duct dimension, see Fig. 1
b_{ij}	elements of matrix $\mathbf{B}$
C	duct contour (m)
c_p	constant pressure specific heat (J/kg K)
$\mathbf{D}$	matrix with elements d_{mj}
Da	Darcy number, K/a^2
D_h	hydraulic diameter $4A/C$ (m)
d_{mj}	elements of matrix $\mathbf{D}$
$\mathbf{E}$	matrix with elements e_{ij}
e_{ij}	elements of matrix $\mathbf{E}$
$F_n(z)$	function, see Eq. (5)
f_i, f_j	basis functions
G	Green's function
h	heat transfer coefficient (W/m^2 K)
$\bar{h}$	average heat transfer coefficient (W/m^2 K)
i, j	indices
K	permeability (m^2)
k_e	effective thermal conductivity
M	μ_e/μ
m, n	indices
N	matrix dimension
Nu_D	Nusselt number, hD_h/k_e
$\overline{Nu}_D$	Nusselt number, $\bar{h}D_h/k_e$
$\mathbf{P}$	matrix having elements p_{mi}
Pe	Peclet number, $\rho c_p aU/k_e$
Pr	Prandtl number, $\mu c_p/k_e$
p	pressure, Pa
p_{mi}	elements of matrix $\mathbf{P}$
Re_D	Reynolds number, $\rho UD_h/\mu_e$
S	volumetric heat source (W/m^3)
T	temperature (K)
T_i	temperature at $x = 0$ (K)
U	average velocity (m/s)
$\overline{U}$	average value of $\bar{u}$
u	velocity (m/s)
$\bar{u}$	$\bar{u} = \mu u/(-a^2\partial p/\partial x)$
x	axial coordinate (m)
$\hat{x}$	$(x/a)/Pe$
y, z	coordinates (m)
$\bar{y}$, $\bar{z}$	y/a and z/a
Greek symbols	
β_m	eigenvalue
γ_m	eigenvalue
θ	dimensionless temperature
λ_m	eigenvalue
μ	fluid viscosity (N s/m^2)
μ_e	effective viscosity (N s/m^2)
ξ	dimensionless coordinate
ρ	fluid density (kg/m^3)
Φ	transformed temperature, Eq. (24)
ψ	eigenfunction
Subscripts	
b	bulk
f	fluid
i	inlet condition
s	source effect
w	wall

The mathematical formulation of temperature for both cases of constant wall temperature and uniform wall heat flux is a necessary part of this presentation. The general solution to each of these two cases has a relatively large number of controlling parameters. Therefore, for brevity of this presentation, the effect of axial conduction is neglected.

2. Mathematical formulations

The basic working relations are the momentum and the energy equations. An exact solution for momentum equation provides the velocity field under a fully developed flow condition and it is available in [9]. The extended weighted residual method, described in [10], is employed in order to determine the temperature distribution from the energy equation. For completeness of this presentation, a brief description of the working relations is to follow.

2.1. Momentum equation

The working relations for the computation of velocity field are widely available in the literature. Their appearance in this paper is for the convenience of identification of the parameters in subsequent numerical analysis. For a laminar flow passing through rectangular passages, Fig. 1(a), with sufficiently high porosity, the entrance length is relatively small [11] and the flow is considered to be hydrodynamically fully developed. Accordingly, the Brinkman momentum equation, as used in Nield et al. [12–14] and Kuznetsov et al. [15] describes the velocity field; that is,

$$\mu_e\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)-\frac{\mu}{K}u-\frac{\partial p}{\partial x}=0 \tag{1}$$

wherein μ_e is the effective viscosity, μ is the fluid viscosity, K the permeability, and the pressure gradient $\partial p/\partial x$ is a constant. By setting $\bar{y} = y/a$, $\bar{z} = z/a$, $M = \mu_e/\mu$, and