Available online at www.sciencedirect.com

SCIENCE DIRECT®

International Journal of Heat and Mass Transfer 48 (2005) 4835–4845

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Dual mixed convection flows in a vertical channel

A. Barletta [a], E. Magyari [b,*], B. Keller [b]

[a] Dipartimento di Ingegneria Energetica, Nucleare e del Controllo Ambientale (DIENCA), Università di Bologna, Via dei Colli 16, I-40136 Bologna, Italy

[b] Chair of Physics of Buildings, Institute of Building Technology, Swiss Federal Institute of Technology (ETH) Zürich, CH-8093 Zürich, Switzerland

Received 29 April 2004; received in revised form 29 April 2005
Available online 15 August 2005

Abstract

The classical problem of the fully developed mixed convection flow with frictional heat generation in a vertical channel bounded by isothermal plane walls having the same temperature is revisited in this paper. The existence of dual solutions of the local balance equations is pointed out. They are either columnar upflows or cellular down–up–down flows. Below a maximum value Ξ_{max} of the governing parameter $\Xi = Ge\,Pr\,Re$ (the product of the Gebhart, Prandtl and Reynolds numbers), for any given Ξ a pair of different solutions occurs. The value Ξ_{max} corresponds to a maximum value of the Reynolds number above which no laminar solution can be found. At this maximum value, the two solution branches bifurcate from each other. In the neighborhood of the bifurcation point Ξ_{max} even small perturbations can cause transitions from one flow regime to the other. In the paper, the mechanical and thermal characteristics of the dual flow regimes are discussed in detail both analytically and numerically.

Keywords: Mixed convection; Vertical channel; Dual solutions; Bifurcation point; Viscous dissipation; Heat transfer

1. Introduction

Buoyancy induced flows in ducts deserve wide attention mainly for their engineering applications in several thermal control devices ranging from electronics to nuclear plants. Indeed, in passive or semi-passive thermal control systems, either purely free convection flows or mixed convection flows are involved.

Obtaining analytical solutions for mixed convection problems in vertical or inclined ducts has been the subject of several papers in the latter decades [1–5]. The importance of such analytical solutions, which refer to laminar fully developed flows, relies on the chance to obtain non-trivial benchmarks to test the reliability of numerical codes developed for more complex geometries or for non-parallel flows. Moreover, analytical solutions are often an opportunity to inspect the internal consistency of the mathematical models and of the approximations adopted, as well as to develop new theoretical results. For instance, in Ref. [5], a novel criterion to choose the reference temperature when adopting the Boussinesq approximation in duct flows has been proposed.

The theoretical investigations on fully developed mixed convection in vertical or inclined ducts are often devoted to a description of the changes on the velocity profiles induced by buoyancy as well as to the

* Corresponding author. Tel.: +41 01 6332867; fax: +41 01 6331041.
E-mail address: magyari@hbt.arch.ethz.ch (E. Magyari).

0017-9310/$ - see front matter
doi:10.1016/j.ijheatmasstransfer.2005.05.036

Nomenclature

A_n	dimensionless coefficients, Eq. (28)
B_n	dimensionless coefficients, Eq. (33)
Br	Brinkman number, $= \mu U_m^2/[k(T(0)-T_0)]$, Eq. (45)
c_p	specific heat at constant pressure
g	acceleration due to gravity
Ge	Gebhart number, $= 4Lg\beta/c_p$, Eq. (16)
h	dimensionless heat transfer coefficient, Eq. (44)
k	thermal conductivity
L	channel half-width
p	pressure
P	difference between the pressure and the hydrostatic pressure
Pr	Prandtl number, $= \mu c_p/k$, Eq. (16)
Re	Reynolds number, $= 4LU_m/\nu$, Eq. (16)
Re_{max}	maximum allowed value of Re
T	temperature
T_0	wall temperature
T_r	reference temperature
u	dimensionless velocity, $= U/U_m$, Eq. (16)
U	X-component of the fluid velocity
U_m	mean fluid velocity, Eq. (10)
$\mathbf{V}$	fluid velocity vector
X, Y	Cartesian coordinates
y	dimensionless transversal coordinate

Greek symbols

α	dimensionless parameter, $= d\phi/dy\|_{y=0}$, Eq. (26b)
β	coefficient of thermal expansion
ϕ	dimensionless velocity gradient, $= (\Xi/16)du/dy$, Eq. (21)
λ	dimensionless parameter, $= -(1/\Xi)d\phi/dy\|_{y=1}$, Eq. (42)
μ	dynamic viscosity
ν	kinematic viscosity, $\nu = \mu/\rho$
ρ	mass density evaluated at the reference temperature
Ξ	dimensionless parameter, $= Ge\,Pr\,Re$, Eq. (16)
Ξ_{max}	maximum allowed value of Ξ

determination of the conditions for the onset of flow reversal (crossover from a columnar to a cellular flow). Indeed, the flow reversal phenomenon arises when buoyancy forces are so strong that there exists a domain within the duct where the local fluid velocity has a direction opposite to the mean fluid flow. These studies are often based on the assumption that the effect of viscous dissipation in the fluid is negligible. This assumption holds whenever the fluid has a sufficiently high thermal conductivity, a sufficiently small Prandtl number and sufficiently high wall heat fluxes are present. On the other hand, other theoretical investigations have been devoted to the analysis of the interplay between the effect of viscous dissipation and the effect of buoyancy [6–14]. These theoretical studies present either analytical or numerical solutions of the local momentum balance equations under the Boussinesq approximation. From a mathematical point of view, viscous heating is represented by a non-linear term (the dissipation function) in the local energy balance equation. Non-linearities due both to inertia and to viscous heating are usually neglected when studying fully developed mixed convection flows in vertical channels. As a consequence, the analysis of such flows allows for a straightforward analytical determination of the velocity and temperature profiles. However, when viscous heating is present, the solutions to be determined are less simple and analytical methods based either on perturbation expansions or on non-linear extensions of the Frobenius method are needed. Perturbation solutions are obtained in Refs. [7,8,10–12] with reference to channel flows. The boundary conditions considered on the walls of the channel are either uniform temperature or uniform heat flux. As it is well known, the latter boundary conditions imply, in the fully developed region, a linearly varying wall temperature in the streamwise direction (when the viscous dissipation is neglected). Dealing with non-linear governing equations may imply non-uniqueness of the solution for a given set of boundary conditions. Examples of dual solutions have been discussed in Refs. [14,15] either for a clear fluid or for a fluid-saturated porous medium.

The aim of the present paper is to analyze combined forced and free flow in the fully developed region of a vertical channel with isothermal walls kept at the same temperature, the fluid properties being assumed as constant. The viscous dissipation effect will be taken into account. The set of governing balance equations will be reduced to a fourth-order ordinary differential equation for the velocity field which will be solved both analytically and numerically. The analytical method will be based on a power series expansion with respect to the transverse coordinate. It will be shown that dual solutions may arise for a prescribed mass flow rate. Comparisons with another approach to the same problem based on a perturbation method by the first author [10] will be performed.