International Journal of Heat and Mass Transfer 52 (2009) 2824–2833

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Visualization of heat flow due to natural convection within triangular cavities using Bejan's heatline concept

Tanmay Basak [a], G. Aravind [a], S. Roy [b,*]

[a] Department of Chemical Engineering, Indian Institute of Technology Madras, Chennai 600 036, India
[b] Department of Mathematics, Indian Institute of Technology Madras, Chennai 600 036, India

ARTICLE INFO

Article history:
Received 15 April 2008
Received in revised form 6 October 2008
Available online 11 February 2009

Keywords:
Heatlines
Heatfunction
Inverted triangular cavity
Streamlines
Streamfunctions
Natural convection

ABSTRACT

Natural convection and flow circulation within a cavity has received significant attention in recent times. The wide range of applicability of flow inside a cavity (food processing industries, molten metal industries, etc.) requires thorough understanding for cost efficient processes. This paper is based on comprehensive analysis of heat flow pattern using Bejan's heatline concept. The key parameters for our study are the Prandtl number, Rayleigh number and Nusselt number. The values of Prandtl number (0.015, 0.026, 0.7 and 1000) have been chosen based on wide range of applicability. The Rayleigh number has been varied from 10^2 to 10^5. Interesting results were obtained. For low Rayleigh number, it is found that the heatlines are smooth and perfectly normal to the isotherms indicating the dominance of conduction. But as *Ra* increases, flow slowly becomes convection dominant. It is also observed that multiple secondary circulations are formed for fluids with low *Pr* whereas these features are absent in higher *Pr* fluids. Multiple circulation cells for smaller *Pr* also correspond multiple cells of heatlines which illustrate less thermal transport from hot wall. On the other hand, the dense heatlines at bottom wall display enhanced heat transport for larger *Pr*. Further, local heat transfer (Nu_l,Nu_t) are explained based on heatlines. The comprehensive analysis is concluded with the average Nusselt number plots. A correlation for average heat transfer rate and *Ra* has been developed and the range of Rayleigh number is also found, to depict the conduction dominant heat transfer.

1. Introduction

The relevance of buoyancy induced circulations causing transport of heat and mass is significant in various physical systems. Especially, the applicability of natural convection inside triangular enclosure, in wide range of engineering processes from energy related to geophysical and material processing industry is very well known [1–6]. In particular, convective heat transfer is widely used in material processing industries like food industries, molten salt application (e.g. fuel cell technology), molten metal applications, etc. [7,8]. Various other studies involving sterilization, solidification of food, food separation processes and other natural convection based processes have also been reported by earlier researchers [9–13]. The analysis of flow for such systems is important for a complete understanding of the problem. Numerical modeling may be employed to understand and analyze these systems. The advantage of numerical simulations is that the expensive experimental costs can be reduced.

Numerical and experimental studies on natural convection in triangular cavities have received significant attention due to various applications. Poulikakos and Bejan [14] have carried out extensive analysis on natural convection in an attic space. Holtzman et al. [15] and Del Campo et al. [16] did numerical study of natural convection in triangular enclosures. Later, Kent et al. [17] and Omri et al. [18] carried out numerical study on right-angled and isosceles triangular cavities, respectively. Varol et al. [19] did the study of natural convection in a triangular enclosure with flush mounted heater on the wall. Recently, Sieres et al. [20] carried out analysis of convection within a triangular enclosure for cavities with variable aperture. A few other recent investigations on natural convection within triangular cavities for various applications have been carried out by earlier researchers [21–26]. However, a comprehensive analysis on natural convection flows in complex enclosures is yet to appear in the literature. It is essential to study the heat transfer characteristics in complex geometries to obtain optimal design of the processes for improving the product quality.

Although a number of numerical investigations [17–26] has been carried out in triangular cavities, the detailed analysis of heat flow was poorly understood. The motivation for this work arises from the fact that there is a lack on visualization of heat flow to analyze the optimal thermal mixing and temperature distribution within triangular enclosures. In view of various applications of thermal processes, a comprehensive understanding of heat transfer

* Corresponding author.
E-mail addresses: tanmay@iitm.ac.in (T. Basak), sjroy@iitm.ac.in (S. Roy).

0017-9310/$ - see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijheatmasstransfer.2008.10.034

Nomenclature

g	acceleration due to gravity ($\mathrm{m\,s^{-2}}$)
k	thermal conductivity ($\mathrm{W\,m^{-1}\,K^{-1}}$)
L	height of the triangular cavity (m)
N	total number of nodes
Nu	Nusselt number
$\overline{Nu}$	average Nusselt number
p	pressure (Pa)
P	dimensionless pressure
Pr	Prandtl number
R	residual of weak form
Ra	Rayleigh number
T	temperature (K)
T_h	temperature of hot inclined wall (K)
T_c	temperature of cold top wall (K)
u	x component of velocity ($\mathrm{m\,s^{-1}}$)
U	x component of dimensionless velocity
v	y component of velocity ($\mathrm{m\,s^{-1}}$)
V	y component of dimensionless velocity
X	dimensionless distance along x-coordinate
x	distance along x-coordinate (m)
Y	dimensionless distance along y-coordinate
y	distance along y-coordinate (m)

Greek symbols

α	thermal diffusivity ($\mathrm{m^2\,s^{-1}}$)
β	volume expansion coefficient ($\mathrm{K^{-1}}$)
γ	penalty parameter
Γ	boundary
θ	dimensionless temperature
ν	kinematic viscosity ($\mathrm{m^2\,s^{-1}}$)
ρ	density ($\mathrm{kg\,m^{-3}}$)
Φ	basis functions
ψ	dimensionless streamfunction
Π	dimensionless heatfunction

Subscripts

i	residual number
k	node number
l	left wall
r	right wall
t	top wall

and flow circulations within triangular cavities is very much essential for industrial development. Current work attempts to analyze heat transfer, correlations and energy distributions using heatline approach.

The heatline concept was first introduced by Kimura and Bejan [27] and Bejan [28]. Heatline is the best tool to analyze and understand the heat flow in 2D convective transport processes and this concept is similar to streamline which is important to analyze fluid motion. Heatlines represent heatflux lines which represent the trajectory of heat flow and they are normal to the isotherms for conductive heat transfer. It may be noted that heatfunctions are mathematical representations of heatlines and each heatline contour corresponds to constant heatfunction. Various applications using heatlines were studied by Bello-Ochende [29], Costa [30–33], Mukhopadhyay et al. [34,35] and Deng and Tang [36]. Recently, Dalal and Das [37] have used heatline method for the visualization of flow in a complicated cavity. However, a comprehensive analysis on heat flow during natural convection in a triangular cavity with the heatline approach is yet to appear in the literature.

The aim of this paper is to study the circulations and temperature distribution and to analyze the flow of heat due to natural convection in an isosceles right angled inverted triangular enclosure with an aspect ratio of 2:1, involving hot inclined walls and cold top wall. The geometry of this enclosure with boundary conditions is shown in Fig. 1. Numerical results are presented in terms of isotherms, streamlines and heatlines along with the local and average heat transfer rates. Galerkin finite element method with penalty parameter has been used to solve the non-linear coupled partial differential equations of flow and temperature fields. To solve the Poisson equation for streamfunctions and heatfunctions, Galerkin method is also used. It may be noted that Galerkin method has been used to evaluate heatfunction for the first time in this work. The jump discontinuity in Dirichlet type of wall boundary conditions for temperature at the corner points correspond to computational singularities. This problem is tackled by considering the average temperature of the two walls at the corner and keeping the adjacent grid nodes at the respective wall temperature similar to earlier works. We have considered Prandtl number from low to high range (0.015–1000) for fluids of various industrial applications. Typically, $Pr = 0.015$ corresponds to molten metals and $Pr = 988.24$ corresponds to olive oil. Non-orthogonal grid generation is done with iso-parametric mapping as given in Appendix A.

2. Governing equations and solution procedure

2.1. Momentum and energy formulation

The fluid properties are assumed to be constant except the density in the body force term which was determined according to the Boussinesq approximation. This approximation is used in the field of buoyancy driven flows and it is based on the fact that density in the body force term varies linearly with temperature. Under these assumptions, the governing equations for steady two dimensional, laminar, incompressible flows can be written in dimensionless form as:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{1}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + Pr\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right), \tag{2}$$

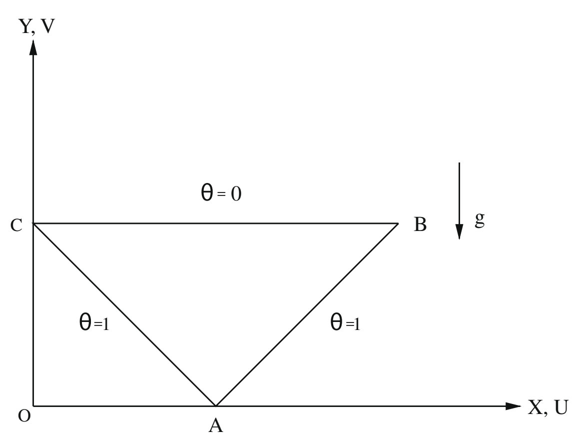

Fig. 1. Schematic diagram of the physical system with the boundary conditions.