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Solar pond modeling with density and viscosity dependent on temperature and salinity

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ABSTRACT

The paper presents a 2D numerical model where the behavior of a salt gradient solar pond (SGSP) is described in terms of temperature, salt concentration and velocity with the fluid density and viscosity dependent on temperature and salt concentration. The discretization of the governing equations is based on the respective weak formulations. The rectangular geometry allows for spectral type Galerkin approximations for which the essential homogeneous boundary conditions can easily be imposed. Taking into account the variation of density and viscosity with temperature and salinity improved the agreement between the numerical and the experimental results.

1. Introduction

A salt gradient solar pond (SGSP) is a basin containing a mixture of water and salt heated by solar radiation and used as an energy storage device. A temperature gradient (hotter at the bottom and cooler at the top) is established and a salt concentration gradient (denser at the bottom and lighter at the top) is therefore created and supposed to prevent convective motions that would otherwise promote the return of the stored energy to the outside ambient and thus destroying the pond's very purpose. A double diffusion process occurs where the temperature and salinity fields make opposite contributions to the fluid density.

There have been several attempts for the numerical solution of the governing equations. For example, Hull [1], Hawlader and Brinkworth [2] and Rubin et al. [3] have applied a finite difference method while [4] has used a finite element technique. The pond stability that constitutes one of the key factors governing a SGSP performance has been studied by several researchers who have resorted in most cases to the linear perturbation theory, see in particular [5–8] and [9]. The results obtained from these studies have provided important information regarding the onset of the instabilities as well as the existence of several possible stable or unstable states that may arise.

Weinberger [10] was the first to give a mathematical formulation of the behavior of a salinity gradient solar pond, analyzing among other things the absorption of the solar radiation by the brine solution, the losses to the atmosphere and to the ground and the double diffusion effect. The analytical solution of the partial differential equations for the transient temperature distribution was obtained by superposing the effects of the radiation absorption at the surface, in the body of water and at the bottom.

Meyer [11] developed a numerical model to predict the time dependent behavior of the interface between the convecting and the non-convecting regions of the solar pond. The model utilizes the empirical correlations that describes the heat and the salt fluxes across the interfaces of the pond regions.

Panahi et al. [12] employed a one-dimensional model to simulate the dynamic performance of the salinity gradient solar pond with a finite element technique.

Angeli and Leonardi [13] and [14] investigated the development of salt concentration profiles in a SGSP and studied the salt diffusion and stability of the density gradient. The prediction of the solar pond stability and performance was made by calculating the optimum salinity gradient thickness and its transient behavior taking into account the seasonal changes of both solar radiation and solar pond temperature (see also [15]).

Mansour et al. [16] solved numerically the problem of transient heat and mass transfer and long term stability of a SGSP through a 2D model and a finite volume method.

In theoretical stability studies, the vertical gradients of temperature and salt concentration are usually assumed constant as this facilitates the analysis, see [17–19,11,8,9].

However, in reality the viscosity may depend strongly on temperature and salt concentration, exponentially or even super-exponentially. In the case of solar ponds, where the temperature can range typically from 90 °C at the bottom and 20 °C at the top, the

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Nomenclature

a	scalar field, Eq. (1)
a, b, c, d	fitting parameters, Eq. (16)
C_p	specific heat [J/kg°C]
$\mathbf{g}$	gravity acceleration [$\mathrm{m/s^2}$]
h	convection coefficient, Eq. (3) [w/mk]
L_1, L_2	domain length and height
$\mathbf{n}$	unit vector
p	pressure [$\mathrm{N^2/m}$]
q	flux, Eq. (2)
S	salt concentration [$\mathrm{kg/m^3}$]
T	temperature [K]
$\mathbf{T}$	viscous stress tensor, Eq. (5)
t	time [s]
u	generic scalar field, Eq. (1)
$\mathbf{v}$	velocity field [m/s]
$\mathbf{x} = (x_1, x_2)$	Cartesian co-ordinates

Greek symbols

α_S	salt diffusivity [$\mathrm{m^3/kg}$]
α_T	thermal diffusivity [$\mathrm{K^{-1}}$]
∂	partial derivative
μ	dynamic viscosity [$\mathrm{m^2/s}$]
γ	convection coefficient, Eq. (3) [$\mathrm{W/m^2k}$]
Ω	domain
Γ	boundary of Ω
ν	kinematic viscosity[$\mathrm{m^2/s}$]
ψ	stream function
θ	dimensionless temperature
ω	dimensionless concentration
ρ	fluid density[$\mathrm{kgm^3}$]
σ	coefficient, Eq. (1)
∇	gradient
$\nabla\cdot$	divergence

Subscripts

1, 2, 3, 4	*i*th face of the domain Ω
amb	ambient
0	initial
f	final
S	salt
T	temperature

Superscripts

$\dot{y}$	time derivative
$\mathbf{v}^{\mathrm{T}}$	transpose

viscosity can vary by one order of magnitude and in many industrial and geophysical applications even much more. In stability studies this implies the base state to depart from constant temperature and salinity gradients. Concerning the case of linear stability, perturbations are assumed to be infinitesimal and then they act with constant viscosity over a base state calculated with a variable viscosity. However, if a full nonlinear analysis is envisaged, the perturbations are no longer infinitesimal and the viscosity variation has to be dully accounted for as was shown both theoretically and experimentally by [20] in the context of a Rayleigh–Bénard problem with glycerol. The observations of [21] show that in a real solar pond the salt gradient is far from constant. In [22] the effect of a constant temperature gradient but a variable vertical salt gradient on the stability of a fluid layer was considered. An experimental programme to assess the various configurations at the onset of convection in the presence of temperature dependent viscosity was carried out by [23].

A linear stability study with variable fluid properties and a nonlinear basic salt concentration was presented in [24] for a horizontally infinite fluid layer subject to small perturbations. In [25,26] the effect of a exponentially temperature dependent viscosity in natural convection for high or infinite Prandtl number is assessed and comparisons with the case of constant viscosity are presented. The control of a SGSP to ensure successful year round operation was studied in [27] employing a one-dimensional model and is typical of the practical difficulties facing a realistic modeling of such devices.

The present paper considers a rectangular cavity filled with either glycerin or a mixture of water and salt (sodium chloride, NaCl) heated at the bottom. The fluids are treated as newtonian incompressible, heat conducting according to Fourier's law and the salt diffusion obeys Fick's law and are subject to a uniform gravitational field. The density is given by the usual linear Boussinesq type approximation and the viscosity by a nonlinear function of the temperature and salinity.

The discretization of the governing equations is based on the respective weak formulations. The rectangular geometry allows for spectral type approximations for which the essential homogeneous boundary conditions can easily be imposed. This choice of method is justified not as much from the outstanding accuracy spectral methods can achieve but rather to obtain a moderate accuracy employing instead a modest number of spatial modes which nevertheless may prove to be adequate for SGSP modeling.

The numerical model developed simulates the three zones that characterize a SGSP attempting to capture the boundary zones behavior by using a non-uniform nodal distribution (Gauss–Legendre–Lobatto nodes).

1.1. Notation

A cartesian coordinate system is employed throughout with position given by $\mathbf{x} = (x_1, x_2)$ and time is denoted by t.

The domain for the examples in Section 4 is a rectangle $\Omega = [0, L_1] \times [0, L_2]$ as depicted in Fig. 1. Its boundary is $\partial\Omega = \bigcup_{i=1}^{4} \Gamma_i$, where the Γ_i are the faces (Γ_1 is on the plane $x_1 = 0$, Γ_2 on the plane $x_1 = L_1$, Γ_3 on the plane $x_2 = 0$ and Γ_4 on the plane $x_2 = L_2$).

2. The governing equations

2.1. The diffusion equations

The two diffusion equations for the temperature T and for the salt concentration S are of the following type

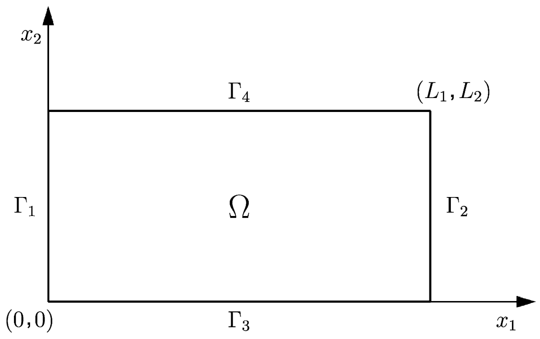

Fig. 1. Geometry and notation.