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Flow and heat transfer in a power-law fluid over a stretching sheet with variable thermal conductivity and non-uniform heat source

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ABSTRACT

In this paper the flow of a power-law fluid due to a linearly stretching sheet and heat transfer characteristics using variable thermal conductivity is studied in the presence of a non-uniform heat source/sink. The thermal conductivity is assumed to vary as a linear function of temperature. The similarity transformation is used to convert the governing partial differential equations of flow and heat transfer into a set of non-linear ordinary differential equations. The Keller box method is used to find the solution of the boundary value problem. The effect of power-law index, Chandrasekhar number, Prandtl number, non-uniform heat source/sink parameters and variable thermal conductivity parameter on the dynamics is analyzed. The skin friction and heat transfer coefficients are tabulated for a range of values of said parameters.

1. Introduction

The study of laminar boundary layer flow and heat transfer in a non-Newtonian fluid over a stretching sheet, issuing from a slit, has gained tremendous interest in the past two decades. A great number of investigations concern the boundary layer behavior on a stretching surface and this is important in many engineering and industrial applications. Flow due to stretching sheet is often encountered in extrusion processes (Fig. 1) where a melt is stretched into a cooling liquid. Apart from this, many metallurgical processes including chemical engineering processes involve cooling of continuous stripes or filaments by drawing them into a cooling system. The fluid mechanical properties desired for the outcome of such a process would mainly depend on the rate of cooling and stretching rate. So, one has to pay considerable attention in knowing the heat transfer characteristics of the stretching sheet as well.

In view of many such applications (see [13]) Crane [1] initiated the analytical study of boundary layer flow due to a stretching sheet. The velocity of the sheet was assumed to vary linearly with the distance from the slit. The work of Crane was subsequently extended by many authors to Newtonian/non-Newtonian boundary layer flow with various velocity and thermal boundary conditions; see, for example, Gupta and Gupta [2], Chen and Char [3], Grubka and Bobba [4], Chiam [5,19,20], Andersson et al. [8,12,31], Siddheshwar and Mahabaleshwar [13], Abel et al. [21,27,28], Liao [29,30], Rajagopal et al. [32] and references therein.

Gupta and Gupta [2] investigated heat transfer from an isothermal stretching sheet with suction/blowing effects. Chen and Char [3] extended the works of Gupta and Gupta [2] to that of a non-isothermal stretching sheet. Grubka and Bobba [4] carried out heat transfer studies by considering the power law variation of surface temperature. Chiam [5] investigated the magnetohydrodynamic heat transfer from a non-isothermal stretching sheet. These studies concern only Newtonian fluids. However, most of the practical situations demand for fluids that are non-Newtonian in nature which are extensively used in many industrial and engineering applications. Acrivos et al. [6] investigated momentum and heat transfer in laminar boundary layer flow of non-Newtonian fluids past external surfaces. Schowalter [7] applied boundary layer theory to study flow of power-law pseudo-plastic fluids and obtained similar solutions. Andersson et al. [8] studied the flow of a power-law fluid over a stretching sheet. Mahmoud and Mahmoud [9] obtained analytical solutions of hydromagnetic boundary layer flow of a power-law fluid past a continuously moving surface. Hassanien et al. [10] investigated the flow and heat transfer in a power-law fluid over a non-isothermal stretching sheet with suction/injection.

An electrically conducting cooling fluid flow can be regulated by an external magnetic field and thereby the heat transfer rate can also be controlled. With this point of view Sarpakaya [11] has investigated the effect of magnetic field on flow of non-Newtonian fluid. Andersson [12] examined the influence of uniform magnetic field on the motion of an electrically conducting viscoelastic fluid over a stretching sheet. Siddeshwar and Mahabaleshwar [13] studied the influence of magnetic field on the flow and heat transfer in a viscoelastic fluid in the presence of uniform heat source and thermal radiation. Abo-Eldahab and Salem [14] studied the influence of

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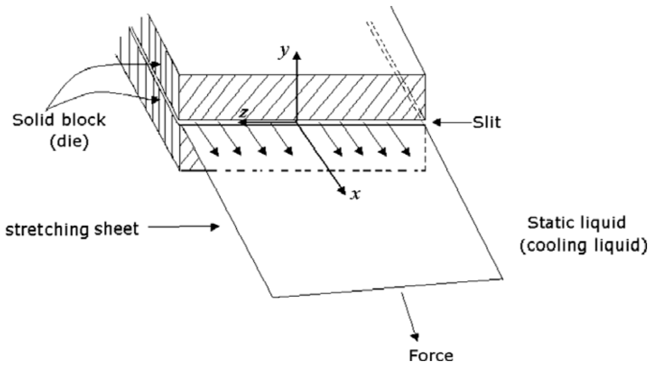

Fig. 1. Schematic of an extrusion process [13].

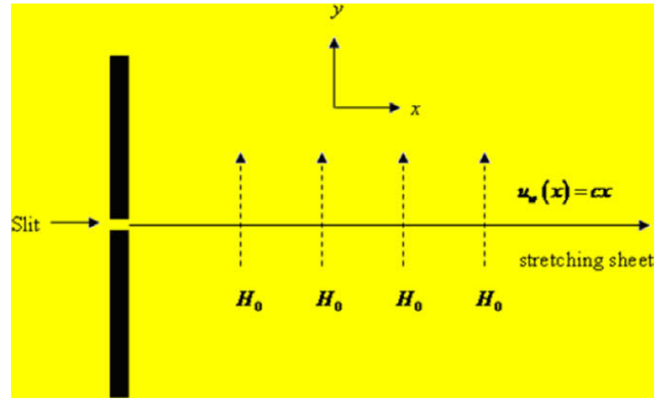

Fig. 2. Schematic of a two-dimensional stretching sheet problem.

transverse magnetic field on the flow and heat transfer of an electrically conducting power-law fluid over a stretching sheet with a uniform free stream. An excellent work on magnetohydrodynamic stretching sheet problem involving a power-law fluid has been reported by Liao [30] using the homotopy based analytical method.

As the study of heat source/sink effect on heat transfer is very important in view of several physical problems, Vajravelu and Rollins [15] and Vajravelu and Nayfeh [16] studied flow due to a stretching surface and heat transfer in presence of uniform heat source/sink (temperature-dependent heat source/sink). Abo-Eldahab and El-Aziz [17] included the effect of non-uniform heat source/sink (space- and temperature-dependent heat source/sink) with suction/blowing. But these works are confined to viscous fluids only. Recently, Abel et al. [27,28] extended the work of Abo-Eldahab and El-Aziz [17] to that of a viscoelastic fluid.

The above-cited works concern constant physical properties for the cooling liquid, but practical situations demand for physical properties with variable characteristics. Thermal conductivity is one such property, which is assumed to vary linearly with the temperature [18]. Chiam [19,20] considered the effect of variable thermal conductivity on heat transfer. Abel et al. [21] have studied the effect of variable thermal conductivity on the MHD boundary layer viscoelastic fluid flow with temperature-dependent heat source/sink, in presence of thermal radiation and buoyancy force.

Motivated by all these works we propose to investigate the effects of variable thermal conductivity, non-uniform heat source on the flow and heat transfer in an electrically conducting power-law fluid over a stretching sheet, in presence of an external transverse magnetic field. In studying the heat transfer characteristics, two different types of boundary conditions are considered.

2. Mathematical formulation

We consider the steady two-dimensional flow of an incompressible, electrically conducting, non-Newtonian power-law fluid obeying Ostwald-de Waele model over a flat impermeable stretching sheet. The flow is generated by the action of two equal and opposite forces along the x-axis and the sheet is stretched with a velocity that is proportional to the distance from the origin (Fig. 2). The flow field is subjected to a transverse uniform magnetic field of strength H_0 and it is assumed that the induced magnetic field is negligibly small (small magnetic Reynolds number limit).

The non-Newtonian fluid model used for the present analysis is the two-parameter power-law model of Ostwald-de Waele with the parameters defined by Bird et al. [22]:

$$\tau = \left(K\left|\sqrt{\frac{\Delta.\Delta}{2}}\right|^{n-1}\right)\Delta, \tag{1}$$

where τ is the stress tensor, Δ is the rate of deformation of symmetric tensor, K is the consistency coefficient, and n is the power-law index. The above power-law model represents Newtonian fluid when $n = 1$, with the dynamic coefficient of viscosity K. If $n < 1$ the fluid is said to be pseudo-plastic (shear thinning fluids) and if $n > 1$ it is called dilatant (shear thickening fluids). The shear stress component of the stress tensor for power-law fluid takes (see [22]) the following form:

$$\tau_{xy} = K\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}. \tag{2}$$

Now, the boundary layer equations governing the flow and heat transfer in a power-law fluid over a stretching sheet, assuming that the viscous dissipation is negligible, are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{3}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial \tau_{xy}}{\partial y} - \frac{\sigma H_0^2 u}{\rho}, \tag{4}$$

$$u\frac{\partial t}{\partial x} + v\frac{\partial t}{\partial y} = \frac{\partial}{\partial y}\left(\frac{k}{\rho C_p}\frac{\partial t}{\partial y}\right) + \frac{q'''}{\rho C_p}, \tag{5}$$

where u and v are the velocity components along x and y directions, respectively, t is the temperature of the fluid, ρ is the density, σ is the electrical conductivity of the fluid, τ_{xy} is the shear stress given by (2), C_p is the specific heat at constant pressure, k is the thermal conductivity which is assumed to vary linearly with temperature and it is of the form, $k = k_\infty\left[1 + \varepsilon\left(\frac{t-t_\infty}{t_w-t_\infty}\right)\right]$ with ε being a small parameter. The non-uniform heat source/sink q''' is modeled as (see [17])

$$q''' = \frac{\rho k u_w(x)}{xK}[A^*(t_w - t_\infty)f' + (t - t_\infty)B^*], \tag{6}$$

where A^* and B^* are the coefficients of space- and temperature-dependent heat source/sink, respectively. Here we make a note that the case $A^* > 0, B^* > 0$ corresponds to internal heat generation and that $A^* < 0, B^* < 0$ corresponds to internal heat absorption.

We have adopted the following two kinds of boundary heating:

(i) prescribed power-law surface temperature (PST)

$$u = u_w = cx, \quad v = 0, \quad t = t_w = t_\infty + A\left(\frac{x}{L}\right)^\lambda \quad \text{at } y = 0, \qquad u \to 0, \quad t \to t_\infty \quad \text{as } y \to \infty, \tag{7}$$