Available online at www.sciencedirect.com
ScienceDirect
International Journal of Heat and Mass Transfer 50 (2007) 1833–1842
International Journal of HEAT and MASS TRANSFER
www.elsevier.com/locate/ijhmt

A general implementation of the **H1** boundary condition in CFD simulations of heat transfer in swept passages

Nathan R. Rosaguti, David F. Fletcher *, Brian S. Haynes

Department of Chemical Engineering, University of Sydney, NSW 2006, Australia

Received 15 May 2006; received in revised form 4 October 2006
Available online 5 December 2006

Abstract

A methodology has been developed to study laminar flow and heat transfer behaviour in periodic non-straight passages with a heat transfer boundary condition of constant axial heat flux and constant peripheral temperature (**H1**). The technique uses Newton iteration to determine the wall temperature distribution required to satisfy the **H1** boundary condition. The methodology is validated for hydrodynamically developed and thermally developing flow, as well as for hydrodynamically and thermally developed flow in straight ducts with various cross-sections. The methodology is extended to study fully developed flow in a periodic serpentine channel, consisting of a number of bends and straight sections, with a semi-circular cross-section. The results show the existence of a non-monotonic temperature distribution along the serpentine channels which exists because increased rates of heat transfer at bends lead to reductions in the local wall temperature in order to maintain a constant axial heat flux. Hot spots within the passage cross-section, typical of the **H2** boundary condition, are removed in the **H1** case.

Keywords: **H1** thermal boundary condition; Forced convection heat transfer; CFD; Constant section ducts

1. Introduction

The growing importance of micro-scale heat exchangers for use in chemical plants of the future and electronic cooling has led to the need to understand laminar flow heat transfer in the periodic passages of heat exchangers. Whilst the specification of hydrodynamic boundary conditions is straightforward, the choice of thermal boundary conditions is more complex. The physical situation is conduction of heat from the hot channel into a substrate and subsequently to a cold channel. The relatively high thermal conductivity of many substrate materials means that axial conduction around the tube walls tends to remove local "hot spots".

Theoretical investigations mostly consider the boundary conditions of uniform wall heat flux and uniform wall temperature (the **H2** and **T** boundary conditions, respectively [1]) due to the relative ease of their implementation in numerical models – Refs. [2–7] are examples of such implementations for a variety of flow geometries. However, these boundary conditions fail to include the important role of the substrate material. The **H1** boundary condition, defined as constant axial wall heat flux and constant peripheral wall temperature, is often a more physically representative boundary condition that goes some way to including the role of the surrounding wall material without moving to a full conjugate heat transfer solution that treats multiple channels and the matrix material in detail [8].

Previous studies of the **H1** boundary condition have been limited mainly to straight ducts [1,9–14]. The compilation of Shah and London [1] provides a database of friction factors and Nusselt numbers for various geometry cross-sections, as well as a thorough explanation of thermal boundary conditions. Table 1, adapted from Shah and London [1], shows the significant differences in Nusselt numbers for fully developed laminar flow in straight ducts of selected cross-sections under the **H1**, **H2**, and **T**

* Corresponding author. Tel.: +61 2 93514147; fax: +61 2 93512854.
E-mail address: d.fletcher@usyd.edu.au (D.F. Fletcher).

0017-9310/$ - see front matter © 2006 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijheatmasstransfer.2006.10.009

Nomenclature

c_p	heat capacity [J kg^{-1} K^{-1}]
d	passage diameter [m]
d_h	hydraulic mean diameter [m]
f	Fanning friction factor [–]
h	heat transfer coefficient [W m^{-2} K^{-1}]
$\mathbf{J}$	Jacobian matrix [W m^{-1} K^{-1}]
k	thermal conductivity [W m^{-1} K^{-1}]
L	serpentine half wavelength [m]
$\dot{m}$	mass flow rate [kg s^{-1}]
N	number of axial locations at which ith column of the Jacobian is evaluated [–]
Nu	Nusselt number $= hd_h/k$ [–]
P	cross-section perimeter [m]
Q_W	total heat transfer rate to the channel [W]
q''	wall heat flux per unit area [W m^{-2}]
q'	wall heat flux per unit length [W m^{-1}]
Pe	Peclet number $= Re \cdot Pr$ [–]
Pr	Prandtl number $= c_p\mu/k$ [–]
R_c	radius of curvature [m]
Re	Reynolds number $= \bar{u}_s \rho d_h/\mu$ [–]
s	axial location along the channel [m]
S	length of the channel [m]
T	temperature [K]
$\overline{T}_B$	bulk mean temperature [K]
ΔT_S	temperature rise along one period of the channel [K]
u_s	velocity in the direction of s [m s^{-1}]
$\mathbf{v}$	velocity [m s^{-1}]
$w(p)$	relative dilatation of the channel at location p [–]

Greek symbols

θ	non-dimensional temperature [–]
μ	dynamic viscosity [Pa s]
ρ	fluid density [kg m^{-3}]

Subscripts

B	bulk
H1	**H1** thermal boundary condition
H2	**H2** thermal boundary condition
M	mean
P	perimeter
W	wall
x, y	(x, y) location on a cross-section

Table 1
Friction factors and Nusselt numbers for fully developed flow in ducts of various cross-sections for different boundary conditions

Cross-section	fRe	Nu_{T}	$Nu_{\mathbf{H1}}$	$Nu_{\mathbf{H2}}$
Circular	16	3.657	4.364	4.364
Square	14.227	2.976	3.608	3.091
Semi-circular	15.767	3.323[a]	4.089	2.923

Values (except[a]) taken from [1].
[a] Taken from [23].

boundary conditions. For non-circular passages, $Nu_{\mathbf{H1}}$ is larger than $Nu_{\mathbf{H2}}$ or $Nu_{\mathbf{T}}$ [9]. Morini [10] summarised some of the research conducted into the **H1** boundary condition as applied to straight ducts with a rectangular cross-section, and considered rectangular cross-sectioned ducts with different combinations of adiabatic and heat transferring walls. Ghodoossi and Eğrican [11] extended the application of the **H1** boundary condition in small scale rectangular channels to include wall slip, a micro-scale phenomenon.

The **H1** boundary condition has also been applied to curved passages. It is well-known that laminar flows in curved passages, such as those which occur in coiled pipes, give rise to heat transfer enhancement relative to flow in straight pipes due to their characteristic secondary flows. Flow and heat transfer research into curved ducts is well summarised by Shah and Joshi [15]. These authors suggest that the influence of the wall thermal boundary condition on the Nusselt number for helical channels with non-circular cross-sections is not significant for $Pr \geqslant 0.7$. Fully developed flow and heat transfer for the **H1** boundary condition case was examined by Kalb and Seader [16] for curved channels with circular cross-sections. Bolinder and Sundén [17] summarised the literature on flow in curved channels with square cross-sections, and investigated the effect of finite pitch on flow and heat transfer behaviour in helical channels.

A common feature of the above work is that it deals with fully developed flow in straight or spiral ducts of constant cross-section – these problems are essentially two-dimensional in nature and relatively easy to solve. Once the problem becomes three-dimensional, for example if the flow is thermally and hydrodynamically developing, or if the duct cross-section is not constant, then the solution of the problem can require very large investment into purpose-built models and computer codes, and therefore the use of commercial CFD packages becomes especially attractive. Most of these packages allow simple Dirichlet ($T = T_{\mathrm{wall}}$, a constant everywhere, corresponding to the T boundary condition) and Neumann ($q''_{\mathrm{wall}} =$ constant everywhere, corresponding under many circumstances to the **H2** boundary condition) boundary conditions and can therefore be used straightforwardly for **T** and **H2** boundary conditions. However, the **H1** boundary condition (heat transfer rate per unit of duct length is constant but the local heat flux around the duct may vary since the peripheral wall temperature at a given axial location is constant) cannot be implemented directly.

The present work aims to fill a gap in the literature by describing a generic methodology to implement the **H1** boundary condition in a computational fluid dynamics