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Conduction tree networks with loops for cooling a heat generating volume

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Abstract

Tree-shaped networks are now being considered as small-scale architectures for high-densities in electronics cooling and fuel cells design. This paper documents the optimization of tree-shaped inserts of high thermal conductivity. The new feature is the presence of loops in the tree canopy. Every feature of the tree-with-loops architecture is optimized numerically. Two classes of trees with loops are considered: loops with one size, and loops with two sizes. The performance of trees with loops is compared with that of trees without loops and designs with purely radial inserts. It is shown that dendrites and loops are features that become attractive as scales decrease and complexity increases. In the same direction, the robustness of tree-with-loops architectures increases.

Keywords: Constructal; Tree networks; Loops; Electronics cooling; Conductive inserts

1. Introduction

The progress toward smaller scales in electronics and the cooling architectures for electronics packages puts a progressively higher price on space. This means that at small enough scales convection ceases as a viable solution for cooling: the space that the fluid-filled cooling channels would occupy is just too valuable not to be allocated to electronics. In this limit, the way to channel the generated heat out of each small volume element is by installing appropriately shaped and placed inserts of high-conductivity.

Conductive inserts are architecturally analogous to the classical fins used in convection. They can be viewed as extended surfaces (needles, blades) that invade the medium, and conduct away the stream of generated heat. Just like in the design of classical fins, where compactness has led to assemblies of fins (arrays, bushes), the maximization of heat transfer density in conduction has led to tree-shaped inserts. This technological step coincided with the emergence of constructal theory [1–5], according to which flow structures such as trees are deterministic results of a principle of maximization of flow access in configurations that are free to morph. The literature on tree-shaped flow architectures is expanding rapidly, not only in electronics cooling, but also in fuel cells, heat exchangers, fluid distribution systems, augmentation, chemical reactors, product platform design, etc. [6–26].

In this paper, we examine a new class of tree-shaped structures for conduction cooling: trees with loops. This direction of inquiry is bio-inspired, because "trees with loops" is the basic architecture for leaf venation. We examine how loops and complexity affect the optimized architecture and its maximized global performance and robustness. We also compare the architecture and performance of trees with loops with older tree architectures without loops [27].

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Nomenclature

$A_{0,1}$	area, m^2
A_p	area covered by k_p material, m^2
d	the smallest length scale, m
D	thickness
i	order of tree link or construct
k_0	thermal conductivity of heat generating material, W m^{-1} K^{-1}
k_p	high thermal conductivity, W m^{-1} K^{-1}
$\tilde{k}$	conductivity ratio, k_p/k_0
L	length, m
N	number of spots on the rim touched by k_p blades
q'''	volumetric heat generation rate, W m^{-3}
R_0	disc radius, m
T	temperature, K
x	radial coordinate, m
y	transversal coordinate, m

Greek symbols	
β	angle, rad
ϕ	area ratio, A_p/A
Subscripts	
max	maximum
min	minimum
mm	minimized twice
mmm	minimized three times
opt	optimum
p	high-conductivity path
Superscript	
(~)	dimensionless, Eq. (7)

2. One loop size, one branching level

Consider first the two-dimensional structure with loops of one size shown in Fig. 1. The structure consists of blades of high-conductivity (k_p) embedded in a heat-generating disc of radius R_0 and lower conductivity k_0. There are two blade sizes, (D_0, L_0) and (D_1, L_1), where the L's are lengths and the D's are thicknesses.

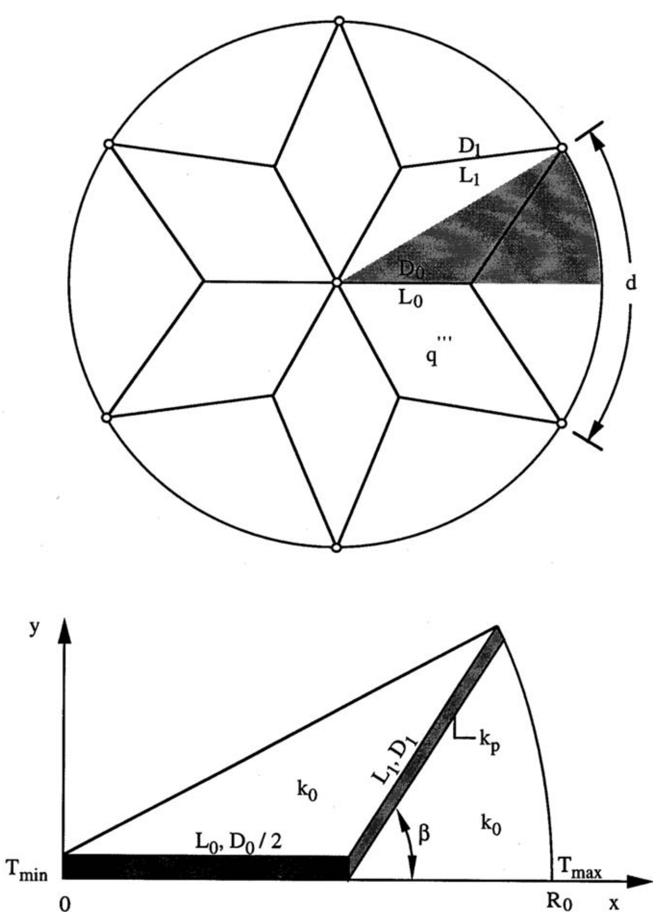

Fig. 1. Disc-shaped body with uniform heat generation, central heat sink, high-conductivity blades, loops of one size, and one level of bifurcation or pairing.

There are N points where the structure touches the rim. The structure is an assembly of trees with only one level of bifurcation, where each L_0 blade is continued by two L_1 blades. The assembly is such that each of the N points on the rim is touched by two of the branched trees. This last feature generates the loops, which are quadrilateral. In this configuration the number of blades that touch the center (n_0) is equal to N.

Heat is generated steadily in the k_0 material, and the volumetric heat generation rate is uniform, q'''. The heat current integrated over the entire disc ($q'''\pi R_0^2$) is collected by the k_p blades and channeled to the center of the disc, which serves as heat sink of temperature T_{min}. The rim is insulated. The highest temperature T_{max} occurs on the rim, in the spots where the radius that is collinear with L_0 intersects the rim. The global thermal resistance of this heat flow structure is expressed in dimensionless form by

$$\tilde{T}_{max} = \frac{T_{max} - T_{min}}{q''' R_0^2 / k_0} \tag{1}$$

The objective is to morph the flow structure in every possible way so that its global resistance $\tilde{T}_{max}$ is minimized. There are several constraints to consider. One is the disc radius R_0. If, in addition, we fix the distance d between two consecutive k_p-points on the rim, then the number of peripheral points is fixed ($N = 2\pi R_0/d$), and so is the area of the sector shown in the lower part of Fig. 1:

$$A_0 = \frac{\pi R_0^2}{2N} \tag{2}$$

Symmetry allows us to focus on heat conduction in the sector of area A_0. The perimeter of this area is insulated,