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Height and stability of laminar plane fountains in a homogeneous fluid

N. Srinarayana [a,*], G.D. McBain [a], S.W. Armfield [a], W.X. Lin [b]

[a] *School of Aerospace, Mechanical and Mechatronic Engineering, The University of Sydney, Sydney, NSW 2006, Australia*
[b] *School of Engineering, James Cook University, Townsville, Qld 4811, Australia*

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Abstract

The behaviour of plane fountains, resulting from the injection of a denser fluid upwards into a large body of a lighter homogeneous fluid, is investigated numerically. The transient behaviour of fountains with a uniform inlet velocity, Reynolds number $Re = 100$, Prandtl number $Pr = 7$, and Froude number $0.25 \leqslant Fr \leqslant 10.0$ is studied numerically. In the present case, the density variation is as a result of temperature difference between the fountain and the ambient fluids. Three distinct regimes are identified; steady and symmetric fountains for $0.25 \leqslant Fr \leqslant 2.0$, unsteady fountains with periodic lateral oscillation for $2.25 \leqslant Fr \leqslant 3.0$, and unsteady fountains with aperiodic lateral oscillations for $Fr \geqslant 4.0$. It is found empirically that the non-dimensional fountain height, z_m, scales differently with Froude number in each of these regimes; in the steady and symmetric region $z_m \sim Fr$, in the unsteady and periodic lateral oscillation region $z_m \sim Fr^{1.15}$ and in the unsteady and aperiodic lateral oscillation region $z_m \sim Fr^{4/3}$. The results are compared with previous numerical and experimental results, where available and are consistent.

Keywords: Plane fountain; Buoyancy dominated flows; Numerical simulation; Scaling; Flapping; Fountain stability

1. Introduction

A fountain forms whenever a fluid is injected upwards into a lighter fluid, or downward into a denser fluid. In the former case the jet penetrates some distance and falls back as a plunging plume around the entering fluid.

Fountains are found in many engineering applications: the heating of a large open structure, such as an aircraft hanger, by large fan-driven heaters at the ceiling level; cooling of turbine blades; cooling of electronic components; the mixing of a two-layer water reservoir with propellers; and the mixing in metallurgical furnaces by gas bubble plumes, to name just a few. Hence, it is important to understand the fundamental physics of such flows.

The behaviour of plane fountains is governed by the Reynolds, densimetric Froude, and Prandtl numbers, defined in the case of a uniform inlet velocity as,

$$Re \equiv \frac{V_{\text{in}} X_{\text{in}}}{\nu}, \quad Fr \equiv \frac{V_{\text{in}}}{\sqrt{g(\rho_{\text{in}} - \rho_\infty)/\rho_\infty X_{\text{in}}}} = \frac{V_{\text{in}}}{\sqrt{g\beta(T_\infty - T_{\text{in}})X_{\text{in}}}}, \quad Pr \equiv \frac{\nu}{\kappa}, \tag{1}$$

where X_{in} is the half-width of the inlet jet. The second expression of the Froude number applies when the density difference is due to the difference in temperature of the fountain and the ambient fluids using the Oberbeck–Boussinesq approximation. It should also be noted that alternatively Richardson number, $Ri = 1/Fr^2$, has been used in the literature [1–3].

For fountains with a relatively large discharge momentum compared to negative buoyancy flux ($Fr \gg 1$) and a

* Corresponding author. Tel.: +61 2 9351 7140; fax: +61 2 9351 7060.
E-mail address: snag3258@usyd.edu.au (N. Srinarayana).

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Nomenclature

C	constant of proportionality, defined in (2)	x	non-dimensional horizontal coordinate
D_H	hydraulic diameter	Y	vertical coordinate
Fr	densimetric Froude number	y	non-dimensional vertical coordinate
f	non-dimensional flapping frequency	z_m	non-dimensional fountain height
g	acceleration due to gravity		
P	pressure	*Greek symbols*	
p	non-dimensional pressure	β	coefficient of volumetric expansion
Pr	Prandtl number	κ	thermal diffusivity
Re	Reynolds number	ν	kinematic viscosity
Ri	Richardson number	ρ	fluid density
T	temperature	τ	non-dimensional time
U	horizontal velocity	θ	non-dimensional temperature
u	non-dimensional horizontal velocity		
V	vertical velocity	*Subscripts*	
v	non-dimensional vertical velocity	in	variable index at the source
X	horizontal coordinate	∞	variable index of the ambient

large Reynolds number, the flow becomes turbulent close to the source. Turner [4] in his experiments observed that the velocity decreased with height and the whole fountain broadened, came to rest and fell back, until it settled down to a nearly steady state, with the top at a lower height than that attained by the first pulse, an upflow in the centre and a downflow surrounding the upflow. There was exchange of fluid between the up and the down currents, and the mixing of the upflow with descending fluid rather than the stationary environment accounted for the smaller fountain height in the steady state. The experiments revealed that the fountain height of a turbulent fountain does not remain constant, but oscillates with time.

Baines et al. [5] obtained an analytical scaling:

$$z_m = CFr^{4/3}, \tag{2}$$

for a plane turbulent fountain, if the source size is small compared with the height of the resulting fountain. Baines et al. [5] conducted a series of experiments on plane fountains and found that $C = 0.65$. However, Campbell and Turner [6] obtained $C = 1.64$–1.97 from their experiments on plane turbulent fountains. Zhang and Baddour [7] studied the effect of mass flux, momentum flux and buoyancy flux on the properties of plane turbulent fountains experimentally by using two different models. The first model (virtual source model) applied the concept of virtual origin proposed by Morton [8] and the second model (zero-entrainment model) ignored the turbulent entrainment. For $Fr < 6.5$, their virtual source model gave,

$$z_m = (2.0 - 1.12Fr^{-2/3})Fr^{4/3}, \tag{3}$$

and their zero-entrainment model gave,

$$z_m = 0.71Fr^2. \tag{4}$$

They used scaling equation (2) for large Froude number experiments ($Fr \geqslant 10$) and obtained $C = 2.0$.

Goldman and Jaluria [9] carried out an experimental investigation on plane turbulent fountains by blowing hot air vertically downward into a chamber and obtained $z_m = 5.83Fr^{0.88}$, by regression analysis. Recently, Lin and Armfield [10] investigated the effect of the Reynolds number on the height of plane fountains. They found that for $Re \leqslant 200$ the fountain height was dependent on the Reynolds number with the following scaling:

$$z_m \sim FrRe^{-1/2}. \tag{5}$$

Their numerical investigations [11] demonstrate that for $0.2 \leqslant Fr \leqslant 1.0$, $Re = 200$ and $Pr = 7$ the following relation can be obtained:

$$z_m = 0.2774 + 1.8696Fr. \tag{6}$$

A number of investigations have also been undertaken into axisymmetric (round) fountains. Campbell and Turner [6] gave,

$$z_m = CFr, \tag{7}$$

and obtained $C = 2.46$ [6] from their experiments on turbulent fountains. Turner [4] also found the height of the starting fountain, i.e., the height maximum attained at start-up, to be a factor of 1.43 greater than the steady value [4]. Morton [8] used entrainment equations to quantify the increasing radius, the decreasing buoyancy and the velocity of dense fluid injected upward into a lighter fluid, and obtained $C = 2.05$ analytically. Morton, however, did not consider the effect of the downflow and hence his analysis is only valid before the fountain falls back. Abraham [12] proposed an analytical solution in which he considered the decrease of the vertical flux of a tracer near the top of the fountain, which was not present in the previous study by Morton [8], where a constant vertical flux was assumed, obtaining $C = 2.74$. Mizushina et al. [13], experimentally investigated a jet discharged upwards into an