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Heat transfer in a viscoelastic fluid flow over a stretching surface with heat source/sink, suction/blowing and radiation

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Abstract

A boundary layer problem on heat transfer in a viscoelastic boundary layer fluid flow over a non-isothermal porous sheet, where the flow is generated due to linear stretching of the sheet and influenced by a continuous suction/blowing of the fluid through the porous boundary, has been presented. In the flow region, heat balance is maintained with a temperature dependent heat source/sink, viscous dissipation and thermal radiation. Applying suitable similarity transformations on the highly non-linear momentum boundary layer equation and thermal boundary layer equation several closed from analytical solutions have been derived for non-dimensional temperature and heat flux profiles in the from of confluent hyper geometric (Kummer's) functions and other elementary functions as its special form. Heat transfer analysis has been carried out for two general types of boundary heating processes, namely, (i) prescribed quadratic power law surface temperature (PST) and (ii) prescribed quadratic power law surface heat flux (PHF) for various values of non-dimensional viscoelastic parameter k_1^*, Prandtl number Pr, Eckert number E, radiation parameter N, suction/blowing parameter v_w and source/sink parameter β. Some of the several important findings reported in this paper are (i) the combined effect of Prandtl number Pr, radiation parameter N and suction/blowing parameter v_w has significant impact in controlling the rate of heat transfer to the boundary layer region through the porous stretching sheet and (ii) radiation and suction can be used as means of cooling the viscoelastic boundary layer flow region. Special cases of our results are in excellent agreement with some of the existing work.

Keywords: Heat transfer; Viscoelastic fluid; Stretching sheet; Suction/blowing; Non-isothermal boundary; Radiation and heat generation

1. Introduction

Momentum and heat transfer in a viscoelastic boundary layer over a linear stretching sheet have been studied extensively in the recent past because of its ever-increasing usage in polymer processing industry, in particular in manufacturing process of artificial film and artificial fibers. In some applications of dilute polymer solution, such as the 5.4% solution of polyisobutylene in cetane, the viscoelastic fluid flow occurs over a stretching sheet [1,2]. Rajagopal et al. [2] have studied viscoelastic second order fluid flow over a stretching sheet by solving the momentum boundary layer equation numerically. Troy et al. [3] discussed uniqueness of the momentum boundary layer equation. Subsequently Chang [4] and Rao [5] showed the non-uniqueness of the solution and derived different forms of non-unique solution. All these works do not take into account the heat transfer phenomenon. Siddappa and Abel [6] have presented similar flow analysis without heat transfer in the flow of non-Newtonian fluid of the type Walters' liquid B. Although Lawrence and Rao [7] presented a work on heat transfer in the flow of viscoelastic fluid over a stretching sheet it did not consider viscous dissipation. However, viscoelastic fluid flow being a non-Newtonian fluid flow generates heat by means of viscous dissipation. There is another important aspect, which should also be taken into the account in a situation when there would be a temperature dependent heat source/sink present in the boundary layer region. In order to deal with both the situations Bujurke

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et al. [8] have presented a work on momentum and heat transfer in the second order viscoelastic fluid over a stretching sheet with internal heat generation and viscous dissipation. An exact analytical solution of MHD flow of a viscoelastic Walters liquid B past a stretching sheet has been presented by Andersson [9]. The effects of internal heat generation on heat transfer phenomenon are excluded from their analysis.

A new dimension is added to the study of viscoelastic boundary layer fluid flow and heat transfer by considering the effect of thermal radiation. Thermal radiation effect might play a significant role in controlling heat transfer process in polymer processing industry. The quality of the final product depends to a certain extent on the heat controlling factors. In view of this Raptis and Perdikis [10] analysed viscoelastic flow and heat transfer past a semi-infinite porous plate having constant suction of the fluid in presence of thermal radiation. Viscous dissipation which must be taken into account in the heat transfer analysis of non-Newtonian fluid flow is excluded from this study. Raptis [11] studied boundary layer flow and heat transfer of micropolar fluid past a continuously moving plate with viscous dissipation in the presence of radiation. Raptis [12] has also investigated the viscoelastic fluid flow past a semi-infinite plate taking into consideration of radiation using Rosseland approximation [13] when the free stream velocity and the temperature of the plate are not constant. However, this work does not deal with the situation when there would be a temperature dependent heat source/sink, viscous dissipation and suction/blowing through the porous boundary surface. Kumari and Nath [14] studied radiation effect in a non-Darcy mixed convection flow over a solid surface immersed in a saturated porous medium using Rosseland approximation. However, their study is confined to viscous fluid flow only. Siddheshwar and Mahabaleswar [15] studied MHD flow and heat transfer in a viscoelastic liquid over a stretching sheet with viscous dissipation, internal heat generation/absorption and radiation. This work does not take into account permeable stretching boundary condition.

Hence, in the present study we investigate the effect of thermal radiation on heat transfer in a boundary layer viscoelastic fluid flow over a semi-infinite porous stretching sheet taking into consideration of the viscous dissipation and temperature dependent heat source/sink. Radiation has been accounted in this study using Rosseland approximation [13].

We know that thermal boundary layer equation with viscous dissipation term is a non-homogeneous partial differential equation involving quadratic power of the velocity gradient. To seek a similarity solution of the thermal boundary layer equation, in case of linear stretching problem, we contemplate to deal with quadratic power law thermal boundary conditions for two general cases of boundary heating of the type (i) prescribed power law surface temperature of second degree (PST) and (ii) prescribed power law surface heat flux (PHF) of second degree. Several closed form analytical solutions for the heat transfer characteristics are obtained in the form of confluent hypergeometric function (Kummer's function). Solutions are also obtained in the form of some other elementary functions as the special cases of Kummer's function.

2. Governing basic equations and source and boundary conditions

2.1. Momentum boundary layer equation

Following the postulates of gradually fading memory, Coleman and Noll [16] derived the constitutive equation of second-order fluid flow in the form

$$T = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 \tag{2.1}$$

where T is the Cauchy stress tensor, $-pI$ is the spherical stress due to constraint of incompressibility, μ is the dynamics viscosity, α_1, α_2 are the material moduli. A_1 and A_2 are the first two Rivlin–Ericksen tensors and they are defined as

$$A_1 = (\operatorname{grad} q) + (\operatorname{grad} q)^{\mathrm{T}} \tag{2.2}$$

$$A_2 = \frac{\mathrm{d}A_1}{\mathrm{d}t} + A_1(\operatorname{grad} q) + (\operatorname{grad} q)^{\mathrm{T}} \cdot A_1 \tag{2.3}$$

The model equation (2.1) was derived by considering up to second-order approximation of retardation parameter. Dunn and Fosdick [17] have given the range of values of material moduli μ, α_1 and α_2 as:

$$\mu \geqslant 0, \quad \alpha_1 \geqslant 0, \quad \alpha_1 + \alpha_2 = 0 \tag{2.4}$$

The fluid modeled by Eq. (2.1) with the relationship (2.4) is compatible with the thermodynamics. The third relation is the consequence of satisfying the Clausius–Duhem inequality by fluid motion and the second relation arises due to the assumption that specific Helmholtz free energy of the fluid takes its minimum values in equilibrium. Later on Fosdick and Rajagopal [18] have reported , by using the data reduction from experiments, that in the case of a second-order fluid the material moduli μ, α_1 and α_2 should satisfy the relation.

$$\mu \geqslant 0, \quad \alpha_1 \leqslant 0, \quad \alpha_1 + \alpha_2 \neq 0 \tag{2.5}$$

They also reported that that the fluids modeled by Eq. (2.1) with the relationship (2.5) exhibit some anomalous behaviour. We must mention that second-order fluid, obeying model equation (2.1) with $\alpha_1 < \alpha_2$, $\alpha_1 < 0$ although exhibits some undesirable instability characteristics the second-order approximation is valid at low shear rate [2]. Now in literature the fluid satisfying the model equation (2.1) with $\alpha < 0$ is termed as second-order fluid and with $\alpha > 0$ is termed as second grade fluid [2].

We consider a laminar steady state incompressible viscoelastic second order fluid flow over a porous semi-infinite stretching sheet. The flow is generated as the consequence of linear stretching of the boundary sheet, caused by simultaneous application of equal and opposite forces along x-