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Numerical simulation of a fountain flow on nonstaggered Cartesian grid system

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Abstract

A liquid–air fountain flow due to the downward motion of a rectangular sleeve over a stationary piston is studied in the paper. Two-dimensional incompressible laminar flows are assumed to prevail in both air and liquid regions. A single set of governing equations over the entire physical domain including the liquid, the air, and the liquid–air interface (free surface) is solved with the extended weighting function scheme and the NAPPLE (nonstaggered APPLE) algorithm on a fixed nonstaggered Cartesian grid system. To ensure the required dynamic contact angle, the liquid meniscus near the sleeve wall is corrected by solving the force balance equation with the geometry method. This is equivalent to introducing a slip condition at the contact line, and thus successfully removes the stress singularity. Steady state solution of the velocity and the pressure as well as the shape of the free surface is obtained. The numerical result evidences the existence of a toroidal-like motion on the free surface postulated by Dussan [E.B. Dussan V., Immiscible liquid displacement in a capillary tube: the moving contact line, AIChE J. 23 (1977) 131–133], although it is quite weak and thin. The resulting free surface profile agrees with the existing experimental observation excellently. Influence of the piston on the flow is discussed.

Keywords: Fountain flow; Nonstaggered Cartesian grid system; Dynamic contact angle; Piston effect

1. Introduction

The problem dealing with displacement of one fluid by another immiscible fluid is encountered in nature and many industrial applications such as coating operation, oil recovery, and mold filling process. It is well-known that a fluid entering the region near the advancing interface of two immiscible fluids in a narrow channel decelerates in the flow direction and acquires a transverse velocity to spill outward the wall. Such a flow characteristic was coined with the term “fountain effect” by Rose [1].

Numerical simulation for the fountain flow is a challenging problem because of three principal numerical difficulties. First, there is a stress singularity at the contact line due to the no-slip condition on the wall for both fluids. Second, the free surface profile having an irregular shape is not known. Third, the capillary force arising from the curvature of the free surface and the dynamic contact angle should be precisely evaluated. To remove the stress singularity at the contact line, Dussan V. and co-worker [2,3] and Cox [4] postulated that there should be a region of size l_s around the contact line in which the no-slip condition breaks down. However, the molecular dynamic simulations [5,6] demonstrate that the slipping length is of the order $l_s \approx 0.001 \mu m$ for smooth solid walls having smoothness on the molecular scale. Similarly, the “effective” slipping length should be on the order of the typical period of the random undulations for rough walls [7,8]. Unfortunately, it is not practical to implement such a tiny slipping length in the numerical simulation for the flow field.

Behrens et al. [9] proposed a rolling model for the advancement of the free surface. However, the rolling model poses to an oscillating advancement for the contact line. Moreover, there is a mass loss on the wall. Similar

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Nomenclature

$a_W, a_E, a_S, a_N, a_P, a_R$	weighting factors of the finite difference Eq. (13a)
Bo	Bond number, $(\rho_l - \rho_a)gL^2\gamma$
Ca	Capillary number, $\mu_l U_c/\gamma$
Fr	Froude number, $U_c/\sqrt{gL}$
$h(x,\tau)$	free surface profile
L	inner width of the rectangular sleeve, Fig. 1
P_{ref}	reference pressure, N m^{-2}
p	dimensionless pressure, $(P - P_{ref})/\rho_l U_c^2$
$\hat{p}$	dimensionless pressure, Eq. (4)
Re	Reynolds number, $\rho_l U_c L/\mu_l$
u, v	dimensionless velocity, U/U_c and V/U_c
U_c	reference velocity
V_{wall}	moving speed of the sleeve, m s^{-1}
v_n	dimensionless normal velocity on the free surface
w_f	weighting function, $z/(1-\exp(-z))$
$\hat{w}_f$	extended weighting function, Eq. (17)
(x,y)	coordinate system
x_{joint}	a location near the wall, Fig. 4
y_{ref}	reference altitude
z	grid Peclet number, Eq. (13b)
Greek symbols	
γ	surface tension, N m^{-1}
$\Delta x, \Delta y$	grid meshes
$\Delta\tau$	virtual time step
θ_D	dynamic contact angle
κ	curvature of the free surface
μ^*	dimensionless viscosity, Eq. (6)
ρ^*	dimensionless density, Eq. (5)
σ_{nn}	normal stress on free surface
τ	virtual time
ψ	stream function, Eq. (40)
Subscripts	
a	air
l	liquid

result was obtained by Kim et al. [10] when the rolling model and the VOF (volume-of-fluid) scheme were used to track the moving free surface. For consideration of the mass conservation on the wall, Mavridis et al. [11] modified the rolling model by imposing the kinematic condition with the no-slip hypothesis at the contact line. When applied to a start-up free surface flow, however, the modified rolling model [11] does not allow the contact line to move until the contact angle increases from the static contact angle to 180°. As a result, the dynamic contact angle is always 180° despite of the capillary number. This does not seem physically realistic because the dynamic contact angle should be a strong function of the capillary number as well-recognized in the literature [12].

In an early experiment on the displacement of mineral oil by glycerine in a Plexiglas circular tube of 6.35 mm inner diameter, Dussan V. [13] observed that the glycerine underwent the familiar fountain flow, while the mineral oil contained a toroidal-like motion in a region adjacent to the interface. Based on the finding, Dussan V. [13] postulated the existence of a toroidal-like motion in the region directly above and immediately adjacent to the interface of two immiscible fluids.

The problem of injection mold filling is one of the important applications of the fountain flow in liquid–air system. The literature in the area (e.g. [14–19]) seems restricted to problems without capillary force and body force. Moreover, the inlet velocity is assumed to have a fully developed parabolic profile. Such investigations closure the problem by imposing some "boundary conditions" on the free surface, and thus cannot observe the flow field in the air region. The purpose of the present work is to re-examine the fountain flow in liquid–air system by solving velocity and pressure in both liquid and air regions on a fixed nonstaggered Cartesian grid system. The free surface profile in the wall region is corrected with the required dynamic contact angle to remove the stress singularity at the contact line. In this connection, both capillary force and body force should be taken into account especially in the wall region.

2. Theoretical analysis

In their experiment, Coyle et al. [20] used a constant-speed motor to lower a transparent acrylic sleeve over a stationary aluminum piston that rested on the floor. A Newtonian silicon oil was poured into the region above the aluminum piston with the acrylic sleeve in the position shown in Fig. 1. The inner cross-section of the sleeve was $2L \times 2W = 0.038\ \text{m} \times 0.114\ \text{m}$. The sleeve had a moving speed of only $V_{wall} = 0.002$ m/s while its length was 1.016 m such that the steady-state flow was reached. Coyle et al. [20] found that the flow was essentially two-dimensional when viewed from the narrow side (see Fig. 1). In the present study, this same flow configuration is formulated by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\rho^* Re\left(\frac{\partial u}{\partial \tau} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial \hat{p}}{\partial x} + \frac{\partial}{\partial x}\left(\mu^*\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu^*\frac{\partial u}{\partial y}\right) \tag{2}$$

$$\rho^* Re\left(\frac{\partial v}{\partial \tau} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial \hat{p}}{\partial y} + \frac{\partial}{\partial x}\left(\mu^*\frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu^*\frac{\partial v}{\partial y}\right) \tag{3}$$