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The onset of thermal instability of a two-dimensional hydromagnetic stagnation point flow

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Abstract

The aim of the present paper is to examine the effects of a constant magnetic field on the thermal instability of a two-dimensional stagnation point flow. First, it is shown that a basic flow, described by an exact solution of the full Navier–Stokes equations exists under some conditions relating the orientation of the magnetic field in the plane of motion to the obliqueness of free stream. The stability of the basic flow is then investigated in the usual fashion by making use of the normal mode decomposition. The resulting eigenvalue problem is solved numerically by means of a pseudo spectral collocation method based upon Laguerre's functions. The use of this procedure is warranted by the exponential damping of disturbances far from the boundary layer and the appropriate distribution of the roots of Laguerre's polynomials to treat boundary layer problems. It is found through the calculation of neutral stability curves that magnetic field acts to increase the stability of the basic flow.

Keywords: Thermal instability; Hydromagnetic; Stagnation point; Laguerre's polynomials

1. Introduction

Magnetic fields are used in many practical situations that involve electrically conducting fluids in motion, like a liquid metal, electrolyte or plasma; they may substantially influence the fluid motion by interacting with the electric current induced in the fluid. Magnetic fields are employed, for example, to drive flows, induce stirring, levitation or to control heat transfer and turbulence. In the following, our concern is with the influence of a constant magnetic field on the convective flow near a two-dimensional stagnation point. We refer to this flow as the Hiemenz flow in hydromagnetics. Let us recall that boundary layer flow associated to a free stream impinging perpendicularly on a flat plate was first investigated by Hiemenz [1], who found an exact solution which is named after him. The mathematically attractive feature of the Hiemenz flow in hydromagnetics, unlike that in the classic Hiemenz flow, is that it represents, under some conditions, an exact solution of the incompressible continuity, energy and Navier–Stokes equations. Many aspects of the problem at hand have been discussed in the past [2–4], because of their occurrence in industrial applications such as nuclear engineering in connection with the cooling of reactors, heat exchangers design or cooling of electronic devices. As in many other configurations, magnetohydrodynamic

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mechanisms affect not only the convective motion of the fluid, but also the stability of the flow. We are not aware of any work for this question in spite of its crucial importance.

In the absence of buoyancy and magnetic forces, it seems that this problem were first considered by Görtler [5] who derived the disturbance equations for the Hiemenz flow. These equations were studied by Hammerlin [6] who demonstrated that plane stagnation flow can sustain three-dimensional disturbances. The algebraic decay required at upstream infinity leads to solutions having a continuous spectrum of spanwise wavenumber. Hammerlin's result seems unsatisfactory because a unique eigenvalue would be expected. Kestin and Wood [7] conjectured that the inconclusive nature of Hammerlin's investigation is the result of the Hiemenz's idealization where the normal velocity component remains everywhere proportional to the normal coordinate. In the real case this velocity component starts out with that property but tends continuously to a constant value at upstream infinity. With this modification and by including certain small terms associated with the curvature of the wall, they predict the existence of a disturbance of unique wavelength corresponding to a regularly distributed system of counter rotating vortices. This result agrees well with their experimental observations and is fully consistent with the vortical structures exhibited in the flow visualization studies of Hodson and Nagib [8] and Sadeh and Brauer [9]. The problem was re-examined by Wilson and Gladwell [10], who argued that the remedy given by Kestin and Wood [7] is irrelevant and the correct solution may be derived from the disturbance equations proposed by Görther [5] together with a more stringent boundary condition at infinity. Wilson and Gladwell [10] proposed that the disturbance quantities must decay exponentially far upstream and they found then a discrete wave spectrum corresponding to stable solutions. Lyell and Huerre [11] developed linear and nonlinear analysis by using Galerkin expansion where the trial functions are the eigenfunctions of the potential stagnation point. They showed that the solution calculated by Wilson and Gladwell [10] constitutes the least damped mode of an infinite number of stable modes, and found that the linearly stable flow can be destabilized by disturbances of sufficiently high level. Nonlinear instability of Hiemenz flow was also found by Kerr and Odd [12] who, moreover, gave a new class of steady solutions to the Navier–Stokes equations, consisting in a periodic array of counter rotating vortices with the axes aligned with the streamwise direction. Nonlinearity is in fact one of the multiple mechanisms which may destabilize the stagnation point flow. For example, the introduction of unsteadiness into the mean flow is found by Thompson and Manly [13] to be a source of instability. The effect of blowing and the superposition of a sufficient crossflow in the free stream are also found by Hall et al. [14] to destabilize the stagnation point flow. When buoyancy alone is taken into account, computations of Chen et al. [15] have revealed that thermal excitation generates three-dimensional disturbances when the Rayleigh number exceeds some critical value. These authors found that the critical Rayleigh number and the critical wavenumber are relatively insensitive to the Prandtl number, when defined on the basis of the thermal boundary layer lengthscale. These findings remain qualitatively unchanged when obliqueness of the free stream [16] and the relative direction of buoyancy forces [17] are taken into account.

In the present contribution our main interest is with the interplay between magnetic and buoyancy forces. It will be shown that these mechanisms act to oppose each other; buoyancy is to destabilize the flow whereas magnetic effects are to increase its stability.

2. Analysis

2.1. Equations of fluid motion

We consider an electrically conducting fluid impinging obliquely on a hot flat plate, kept at a constant temperature T_w, and lying in the (x^*, z^*) plane which can be, without loss of generality, considered horizontal. The y^* axis is normal to the plate and pointing towards the flow. The latter is submitted to the action of an external uniform magnetic field **B**, arbitrarily oriented in the (x^*, y^*) plane as shown on Fig. 1. In addition to the usual MHD (magnetohydrodynamic) approximations, we will assume that the magnetic Reynolds number which is a measure of the ratio of magnetic convection to magnetic diffusion, is much less than unity. This indicates that the magnetic field is practically unmodified by the flow, whereas the former can control the latter very strongly on a laboratory scale. Further, we will assume that the induced electric field is negligible (see Appendix A). This corresponds to the case where no energy is added to (or extracted from) the fluid by electrical means. So, the electric term is not included in the relevant equations and only the applied magnetic field plays a role. Moreover, Joule heating and Hall effects are ignored. Hence, making use of the Boussinesq approximation, the equations of mass, momentum and energy conservation may be expressed, in terms of the temperature T and velocity $\mathbf{v}$, as

$$\nabla \cdot \mathbf{v} = 0 \tag{1}$$

$$\mathbf{v_t} + (\mathbf{v} \cdot \nabla)\mathbf{v} + \frac{\nabla p}{\rho} = \nu \nabla^2 \mathbf{v} - \mathbf{g}\beta(T - T_\infty) + \frac{\sigma}{\rho}(\mathbf{v} \wedge \mathbf{B}) \wedge \mathbf{B} \tag{2}$$

$$T_t + (\mathbf{v} \cdot \nabla)T = \frac{\nu}{Pr}\nabla^2 T \tag{3}$$