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Forced oscillation in diffusion flames near diffusive–thermal resonance

H.Y. Wang [a], J.K. Bechtold [b], C.K. Law [a,*]

[a] Department of Mechanical and Aerospace Engineering, Princeton University, Princeton, NJ 08544, USA
[b] Department of Mathematical Sciences, New Jersey Institute of Technology, Newark, NJ 07102, USA

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Abstract

In this work, we carry out a systematic analysis of forced oscillation in planar diffusion flames under weak external forcing. The external forcing is introduced by independently imposing a flow field with small amplitude fluctuations. Employing the asymptotic theory of Cheatham and Matalon, the linear response is first examined. It is shown that when the Damköhler number Da is close to the critical value Da^* corresponding to the marginal state of diffusive–thermal pulsating instability, the imposed velocity fluctuation may induce very large amplitude of flame oscillation as the frequency of velocity fluctuation c approaches c_0, the flame oscillation frequency at the onset of instability. This is a resonance phenomenon between the imposed flow oscillations and the intrinsic flame oscillations that are driven by the diffusive–thermal instability, and hence we refer to this as the diffusive–thermal resonance. The nonlinear near-resonant response is then examined with the Damköhler number Da chosen to be very close to the critical Damköhler number Da^*, and we derive an evolution equation for the amplitude of forced oscillation. Examination of the evolution equation reveals that in most situations, flames with larger Lewis number of fuel, smaller initial mixture strength, and smaller temperature difference between the oxidant and fuel stream are more responsive to the external forcing.

Keywords: Diffusion flame; Forced oscillation; Pulsating instability; Linear response; Nonlinear response

1. Introduction

Flames in practical combustors are subjected to fluctuating flows imposed by the random motion of eddies whose wide spectrum of length and time scales may interact with the flames in very different ways. Since a direct study of the flame response to flow unsteadiness in turbulent combustion is rather complicated, the effect of flow unsteadiness on laminar flames has received considerable attention for its potential application to the fundamental understanding and modeling of turbulent combustion through the concept of laminar flamelets [1].

Unsteady effects on both diffusion and premixed flames have been studied with emphasis on the dynamic response to oscillatory strain rate variations. In particular, results on diffusion flames [2–14] show that the flame response becomes more sensitive to the imposed unsteadiness when the otherwise steady flame is near its extinction limit; whereas the response for flames far from extinction is attenuated monotonically as the frequency of the imposed oscillation increases. Consequently, unsteady flames can withstand higher strain rates at higher frequencies than at lower frequencies. However, there have been relatively few previous theoretical investigations. Strahle [2] studied the convective droplet burning at a stagnation point under the influence of small amplitude sound wave from the free stream. Im et al. [13,14] analyzed the response of counterflow diffusion flames to monochromatic oscillatory strain rates using large activation energy asymptotics, with attention focused on near extinction conditions so that the time scale of the imposed unsteadiness is comparable to that of diffusive transport. The results of Im et al. [13] suggest that the unsteady characteristics of the near-extinction diffusion

* Corresponding author. Tel.: +1 (10) 609 258 5271; fax: +1 (10) 609 258 6233.
E-mail address: cklaw@princeton.edu (C.K. Law).

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Nomenclature

A	amplitude function
c_0	frequency at the onset of intrinsic oscillation
Da	Damköhler number
Da^*	Damköhler number at the onset of instability
h_j^*	excess/deficiency enthalpy for species j, $j = F, O$
Le_j	Lewis number of species j, $j = F, O$
R	modulus of amplitude function A
S_j	leakage function for species j, $j = F, O$
t	fast time
T	temperature
u	response of temperature to external forcing
v	response of fuel mass fraction to external forcing
w	response of oxidant mass fraction to external forcing
x_f	location of flame surface
Y_F	fuel mass fraction
Y_O	oxidant mass fraction

Greek symbols

β	Zeldovich number
δ	reduced Damköhler number
ϕ	initial mixture strength
γ	heat transfer parameter
θ	polar angle of amplitude function A
τ_1, τ_2	slow time variables
ξ_f	location of stoichiometric flame surface
λ_J	$\frac{1}{2}\sqrt{Le_J^2 + 4iLe_J c}$, $J = T, F, O$
λ_J	$\frac{1}{2}\sqrt{Le_J^2 + 4iLe_J c_0}$, $J = T, F, O$
μ_J	$\frac{1}{2}\sqrt{Le_J^2 + 8iLe_J c_0}$, $J = T, F, O$

Subscripts and superscripts

b	basic state quantity
p	particular solution to the flame responses
F	fuel
O	oxidant
$-\infty$	fuel boundary
∞	oxidant boundary
$+$	oxidant side of flame sheet
$-$	fuel side of flame sheet

flame can be significantly different from those in the Burke–Schumann limit.

These earlier studies, however, have not addressed the important issue of resonance. That is, combustion systems may exhibit intrinsic oscillation of different modes and those oscillations may then interact with the imposed flow oscillations so that the flame responses could be significantly different. For example, recent studies have shown that, when the Lewis numbers of the reactants are sufficiently larger than unity, intrinsic oscillations due to the imbalance of thermal and mass diffusions, referred to as the thermal–diffusive pulsating instability, may develop near but prior to extinction, leading eventually to flame quenching [15–18]. Thus, such unstable diffusion flames could extinguish at a larger Damköhler number, denoted as the dynamic extinction Damköhler number, Da^*, than the static extinction Damköhler number, Da_{ext}.

The primary objective of the present study is therefore to analyze the flame response to external forcing coupled with intrinsic flame oscillations. Specifically, we consider the simple geometric configuration of a planar diffusion flame situated in a channel at the interface between a fuel being supplied from below with a velocity field with harmonic fluctuation of small amplitude, and an oxidant diffusing in from a cross-stream above. This configuration eliminates the effect of strain rate so that the flame is only subjected to the unsteadiness of the velocity field. Intrinsic oscillation of the planar diffusion flame due to thermal–diffusive instability is considered. The Lewis numbers for both the fuel and oxidant are assumed to be larger than unity and focus our attention on the flame response near the dynamic extinction limit, Da^*, instead of the static extinction limit, Da_{ext}, considered in previous studies. We carry out a systematic analysis on the linear and nonlinear response of the flame oscillation subjected to small amplitude, harmonic velocity fluctuation by employing the asymptotic theory of Cheatham and Matalon [19]. The linear response shows that the resonance phenomena may occur as the frequency of velocity fluctuation approaches the intrinsic oscillation frequency when the flame is near the stability boundary. The nonlinear near-resonant response is then analyzed by deriving an evolution equation for the amplitude of forced oscillation. The Damköhler number Da and forced frequency c are chosen to be close to Da^* and the intrinsic oscillation frequency, c_0, so that even very weak forcing is able to induce rather large oscillation amplitude. It is shown that by considering the inherent nonlinearity, the flame oscillation exhibits finite amplitude at the resonant condition.

2. Formulation

We consider the simple configuration of a planar flame in a chamber [18,19]. As shown in Fig. 1, the fuel stream is fed from the bottom of the chamber and the oxidant diffuses against the fuel stream from a fast cross-stream at the top of the chamber. We employ the asymptotic theory of Cheatham and Matalon [19] in which the convective–diffusive equations for temperature and fuel and oxidant mass fractions are solved on either side of the flame surface, x_f.