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Theoretical analysis of convective heat transfer of Oldroyd-B fluids in a curved pipe

Mingkan Zhang *, Xinrong Shen, Jianfeng Ma, Benzhao Zhang

Institute of Fluid Engineering, Zhejiang University, Hangzhou 310027, China

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Abstract

Perturbation methods are used to study steady, fully developed flow of Oldroyd-B fluids through a curved pipe of circular cross-section. A perturbation solution up to secondary order is obtained for a small value of curvature ratio. The range of validity of the perturbation method are discussed and chosen carefully. Variations of temperature distribution with *Re* and *We* are discussed in detail in order to investigate the combined effects of the two parameters on temperature distribution. Present studies also show the variations of the heat transfer rate with *Re* and *We*. This study explores many new characteristics of convective heat transfer of a kind of viscoelastic fluid through curved pipes.

Keywords: Convective heat transfer; Oldroyd-B fluid; Perturbation method; Pipe flow

1. Introduction

In this paper, perturbation solutions for heat transfer of viscoelastic fluids in curved pipes are obtained. It is assumed that the fluid flow is steady, hydrodynamically and thermally fully developed, both the wall heat flux and the peripheral wall temperature of one cross-section are uniform (different wall temperature in different cross-section), and, the viscous dissipation is negligible.

Since the initial work by Dean [1,2], more and more attentions have been paid to the mass and heat transfer of Newtonian fluid through curved pipes, not only because of its practical importance in various industrial applications, but also because of physically interesting phenomena caused by the curvature of the pipe. The previous works on heat transfer concerned on the planar curved pipes with a circle cross-section, such as Akiyama and Cheng [3], Patanker et al. [4] and Yang and Chang [5]. Such works indicated that both the efficiencies of convective heat transfer and Nusselt number in curved pipes are much greater than those in straight pipes. Then Garimella and Chdrards [6] investigated the forced convective heat transfer in coiled annular ducts experimentally.

By using numerical method, Yang and Ebadian [7,8] and Choi and Park [9] studied the heat transfer and mixed convection flow in a curved annular-sector duct, respectively. More recently, Chen and Zhang [10,11] extended the former work to the heat transfer in a rotating helical pipe.

Besides Newtonian fluid, Viscoelastic fluids are also widely used in industries. Lots of industrial materials fall into this category, such as solutions and melts of polymers, soap and cellulose solutions, biological solutions, various colloids and also paints, tars, asphalts and glues. The Oldroyd-B model can be found frequently in the field of blowing and extrusion molding as well. However, it's rather surprising to find that, despite its important applications, the flow and heat transfer of viscoelastic fluids in pipes has received much less attention in the monographs than its Newtonian counterpart. Robertson and Muller [12] and Jitchote and Robertson [13] presented the perturbation solutions of flow of Oldroyd-B fluid and second order fluid,

* Corresponding author. Tel.: +86 571 87953220.
E-mail address: wangtwo@zju.edu.cn (M. Zhang).

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Nomenclature

a	radius of the circle cross-section
D	symmetric part of the velocity gradient
$\boldsymbol{e}_r$, $\boldsymbol{e}_\phi$, $\boldsymbol{e}_s$	unit base vectors of the convected coordinates system
$\boldsymbol{e}_1$, $\boldsymbol{e}_2$, $\boldsymbol{e}_3$	unit base vectors of the Cartesian coordinate system
G	axial gradient of $w, G = \partial w/\partial s$
H	axial gradient of $T, H = \partial T/\partial s$
p	pressure
Pr	Prandtl number, $Pr = \eta/\alpha$
Pe	Peclet number, $Pe = RePr$
R	curvature radius
Re	Reynolds number, $Re = \rho a W_0/\eta$
r	radial direction coordinates
s	axial direction coordinates
T, T_w	temperature of fluid and wall
T_b	bulk temperature, $T_b = \int_0^1\int_0^{2\pi} Twr\,\mathrm{d}\phi\,\mathrm{d}r/\int_0^1\int_0^{2\pi} wr\,\mathrm{d}\phi\,\mathrm{d}r$
T_0	characteristic temperature, $T_0 = aPrH$
$\boldsymbol{u}$	vector of velocity
u, v, w	physical velocity components
W_0	characteristic temperature, $W_0 = Ga^2/4\eta$
We	Weissenberg number, $\lambda W_0/a$

Greek symbols

α	thermal diffusivity
η_s, η_p	solvent viscosity and polymeric contribution to the viscosity
η	sum of η_s and η_p
κ	curvature ratio
λ	relax time
ρ	density of the fluid
$\boldsymbol{\tau}$	extra stress tensor
ϕ	angular coordinate
ψ	stream function

Subscripts and superscripts

$*$	dimensional variable
max	maximum value
∇	upper-convected derivative

respectively. Almost at the same time, Fan et al. [14] investigated the comparison between fully developed viscous and viscoelastic flows in curved pipes by using finite element method. In their work, they investigated not only the flow characteristics but the two normal stress differences as well. In the field of heat transfer of viscoelastic fluids, Cho and Harnett [15] analyzed heat transfer of polyacrylamide in Chicago tap water. Then Toh and Ghajar [16] reported thermal entrance region Nusselt values for turbulent flow of two different polyacrylamides in circular tubes experimentally. Recently, Pinho and Oliveira [17] got the analytic solution for forced convection of Phan-Thien–Tanner fluid in straight pipes.

In present work, we have extended the previous analysis of Robertson and Muller to the convective heat transfer problem of fully-developed flow of an Oldroyd-B fluid in a curved pipe. It's quite necessary and useful to investigate this problem. Besides the industrial applications, perturbation solutions of present work can provide valuable tests for the numerical simulation of heat transfer of viscoelastic fluids. Moreover, it is much easier to analyze perturbation solutions than numerical ones. In this paper, the combined effects of viscosity, centrifugal force and elasticity on heat transfer are examined in detail. Many new and interesting conclusions are drawn.

2. Governing equations

The momentum and constitutive equations of present work are from reference to the work of Robertson and Muller. We consider the incompressible Oldroyd-B fluids, for which the extra stress tensor τ^* can be written as

$$\boldsymbol{\tau}^* = \boldsymbol{\tau}^{\mathrm{s}*} + \boldsymbol{\tau}^{\mathrm{p}*}, \tag{1}$$

where $\boldsymbol{\tau}^{\mathrm{s}*}$ and $\boldsymbol{\tau}^{\mathrm{p}*}$ are defined as

$$\boldsymbol{\tau}^{\mathrm{s}*} = 2\eta_\mathrm{s}\boldsymbol{D}^*, \quad \boldsymbol{\tau}^{\mathrm{p}*} + \lambda \overset{\nabla}{\tau}{}^{\mathrm{p}*} = 2\eta_\mathrm{p}\boldsymbol{D}^*. \tag{2}$$

In Eq. (2), η_s, η_p and λ are referred to as the solvent viscosity, polymeric contribution to the viscosity and polymer relaxation time, respectively. The rate of deformation tensor, $\boldsymbol{D}^*$, is the symmetric part of the velocity gradient. The components of $\boldsymbol{D}^*$ relative to a rectangular coordinate system are

$$D_{ij}^* = \frac{1}{2}\left(\frac{\partial v_i^*}{\partial x_j^*} + \frac{\partial v_j^*}{\partial x_i^*}\right). \tag{3}$$

The "∇" notation introduced in (2) denotes the upper-convected derivative, which for an arbitrary second-order tensor $\boldsymbol{S}^*$ with coordinates $\boldsymbol{S}_{ij}^*$ relative to a rectangular coordinate system, is

$$\overset{\nabla}{\boldsymbol{S}}{}_{ij}^* = \frac{\partial \boldsymbol{S}_{ij}^*}{\partial t^*} + v_k^*\frac{\partial \boldsymbol{S}_{ij}^*}{\partial x_k^*} - \frac{\partial v_i^*}{\partial x_k^*}\boldsymbol{S}_{kj}^* - \boldsymbol{S}_{ik}^*\frac{\partial v_j^*}{\partial x_k^*}. \tag{4}$$

In the limit of λ equal to zero, the Oldroyd-B equation reduces to the Newtonian constitutive equation and for vanishing η_s to the upper-convected Maxwell constitutive equation.

Fig. 1 shows the curved pipe and the convected coordinates system (r^*, ϕ, s^*) used in present work. R is the radius of curvature of the pipe and a the radius of the circle cross-section. $\boldsymbol{e}_r$, $\boldsymbol{e}_\phi$, $\boldsymbol{e}_s$ are the unit base vectors of the convected coordinates system (r^*, ϕ, s^*) defined relative to the unit base vectors of the Cartesian coordinate system $\boldsymbol{e}_1$, $\boldsymbol{e}_2$, $\boldsymbol{e}_3$ as