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Numerical study of heat transfer over banks of rods in small Reynolds number cross-flow

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Abstract

This work presents numerical computations of heat transfer over banks of square rods in aligned and staggered arrangements with porosity in the range 0.44–0.98. It is focused on low Reynolds number flows (0.05–40). Two thermal boundary conditions were investigated, namely constant wall temperature and constant volumetric heat source. The effects of bank arrangements and porosity as well as the effects of Prandtl and Reynolds numbers on the Nusselt number are examined. In the case of constant volumetric heat source, the results are approximated with a power equation adapted for the case of low *Re* number flows. This study shows that the thermal boundary condition on the solid surface influences heat transfer when thermal equilibrium is reached in the bank of rods.

Keywords: Bank of rods; Laminar flow; Thermal boundary condition; Thermal equilibrium

1. Introduction

Much work has been done in the past on convective heat transfer in banks of tubes or rods in cross-flow. One of the most extensive reviews in the field of cross-flow heat exchanger is that of Zukauskas [1], who proposed correlations between the Nusselt, Reynolds and Prandtl numbers for various arrangements of cylindrical tube banks. These correlations are available for moderate to high values of the Reynolds number ($1 < Re < 2 \times 10^6$).

On the other hand, banks of rods have very often been used as a geometrical model for low Reynolds number flows through porous media. Spatially periodic models have been considered to compute the permeability of the medium as a function of porosity and Reynolds number [2–10]. These models are very attractive for numerical simulations, since the computations may be restricted to a simple cell extracted from the periodic pattern. The role of finite Reynolds number flow and the deviation due to non-linearities from the original Darcy's law have been extensively discussed in the literature. There are much less numerical works on heat transfer over banks of rods in low Reynolds number cross-flow [11–16]. In the context of porous media, one of the issues is that of local thermal equilibrium of the fluid and the solid matrix constituting the porous medium. This problem is much more complex than the isothermal one, since heat transfer not only depends on the porosity and the Reynolds number, but also on the Prandtl number and on the thermal conditions on the solid surfaces. The range of parameters and boundary conditions found in [11–16] are shown in Table 1.

The objective of our work was to establish a database for the heat exchange coefficient in banks of squared rods with the thermal condition of uniform volume source heating and for low Reynolds number flows. The motivations were twofold. Firstly, the thermal condition in heat exchangers is often neither uniform flux nor uniform temperature heating. The influence of this condition on the heat transfer coefficient is negligible in turbulent flows, but may be significant for low Reynolds number flows,

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Nomenclature

a_{sf}	interfacial surface area per unit length, m
C_d	drag coefficient
C_F	Forchheimer coefficient
c_p	Specific heat at constant pressure, J kg^{-1} K^{-1}
C_r	resistance factor
Da	Darcy number
d	solid element size, m
e	channel width, m
h	convective heat transfer coefficient, W m^{-2} K^{-1}
K	permeability, m^2
K_{app}	apparent permeability, m^2
k	thermal conductivity, W m^{-1} K^{-1}
L	longitudinal and transversal pitch, m
$\dot{M}$	mass flow rate per unit length, kg s^{-1} m^{-1}
Nu_{2e}	Nusselt number based on the mean fluid temperature (Eq. (17))
$Nu_{2e,b}$	Nusselt number based on the bulk fluid temperature
$Nu_{2e,x}$	local Nusselt number based on the bulk fluid temperature
Nu_d	Nusselt number (Eq. (11))
Pe	Peclet number ($=Re_{2e}Pr$)
Po	Poiseuille number
Pr	Prandtl number
q_v	volumetric heat source, W/m^3
Re_{2e}	Reynolds number in a channel (Eq. (18))
Re_d	Reynolds number (Eq. (9))
Re_D	Darcian Reynolds number (Eq. (10))
T	temperature, K
u_D	Darcy velocity, m s^{-1}
x^*	dimensionless distance $\left(=\frac{x}{2e}\frac{1}{Pe}\right)$

Greek symbols

ε	porosity
ρ	density, kg m^{-3}
μ	dynamic viscosity, kg m^{-1} s^{-1}
φ	heat flux density, W m^{-2}
σ_0	dimensionless temperature
θ	dimensionless temperature

Subscripts

b	bulk
D	Darcy
f	fluid
max	maximal
min	minimal
p	pressure
s	solid
w	wall

which are predominant in the field of microheat transfer. It is then important to test the sensitivity of the heat transfer coefficient to the thermal condition for the design of micro-heat exchangers. Additionally, the situation of uniform volume source heating is encountered in experimental works on arrays of cylinders with cross-flow convection where the cylinders are electrically heated at uniformly distributed rate [17].

Secondly, we are developing a numerical model for roughness effects on microchannel flows using a discrete-element method initially proposed by Taylor et al. [18,19] for predicting the rough-wall skin friction and heat transfer coefficient in turbulent flows. This method needs correlations for the drag coefficient and the heat exchange coefficient of a cylinder in two-dimensional cross-flow. Following this approach, Bavière et al. [20] considered a rough-wall consisting in periodically distributed parallelepipeds of square cross-section. They estimated the drag coefficient by using the formula for the drag force on very slender prolate spheroids in creeping flows. Their work was restricted to isothermal flows and is currently being extended to improve the determination of the drag coefficient and to take into account heat transfer in the microchannel. The present paper is therefore devoted to numerical computations of the flow and heat exchange in banks of rods of square cross-section heated by volume

Table 1
Conditions from the literature for heat transfer computations in periodic arrays of rods

References	Rod cross-section	Geometrical arrangement	Re	Pr	Porosity	Thermal conditions
Martin et al. [11]	Cylindrical	Aligned in squared or triangular arrays	3–160	0.72	0.8–0.99	Constant wall temperature or heat flux
Kuwahara et al. [12]	Squared	Staggered	2×10^{-3}–10^3	10^{-2}–10^2	0.36–0.91	Constant wall temperature
Ghosh Roychowdhury et al. [13]	Cylindrical	Aligned or staggered	40–1000[a]	Not given	0.5–0.8	Constant wall temperature
Mandhani et al. [14]	Cylindrical	Staggered	1–500	0.1–10	0.4–0.99	Constant wall temperature or heat flux
Nakayama et al. [15]	Squared	Aligned or cylinders in yaw	10^{-2}–6×10^3	1	0.25–0.875	Constant wall temperature
Saito and de Lemos [16]	Squared	Staggered	4–400	1	0.44–0.9	Constant wall temperature

[a] Re is based on the mean velocity at the minimum cross-sectional area in [13].