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Unsteady heat transfer from an elliptic cylinder

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Abstract

Numerical methods are used to investigate the transient heat transfer from an elliptic cylinder to a steady stream of viscous, incompressible fluid. The temperature of the cylinder is considered spatially uniform but not constant in time. The momentum and heat balance equations were solved numerically in elliptic coordinate system. The solutions span the parameter ranges $5 \leqslant Re \leqslant 40$, $1 \leqslant Pr \leqslant 100$ and axis ratio ε, $0.1 \leqslant \varepsilon \leqslant 0.75$. The computations were focused on the influence of the axis ratio and volume heat capacity ratio on the heat transfer rate.

Keywords: Unsteady heat transfer; Forced convection; Elliptic cylinder; Laminar flow; Finite difference method

1. Introduction

In many industrial applications, where heat loads are substantial and space is limited, the elliptical geometry outperforms the circular geometry. Elliptical cylinders offer less flow resistance and higher heat transfer rates than circular cylinders. In spite of this fact, the heat transfer from an elliptical cylinder is the subject of relatively few theoretical/numerical studies.

The laminar mixed (natural and forced) convective heat transfer from a straight isothermal tube of elliptic cross-section placed in a uniform stream was investigated numerically in [1]. The free stream direction is horizontal and normal to the tube axis and the flow field is two-dimensional. The effects of the Reynolds number, $20 \leqslant Re \leqslant 500$, Grashof number, $0 \leqslant Gr \leqslant 1.25 \times 10^6$, Prandtl number, Pr, axis ratio and the angle of inclination (varying from 0° to 180°) on the heat transfer process were studied. In [2] the two-dimensional steady-state problem of laminar forced convective heat transfer from an isothermal cylinder, elliptic in cross section, inclined to a uniform stream is investigated. Numerical solutions of the Navier–Stokes and energy equations have been obtained for Reynolds numbers, Re, 5 and 20, Prandtl number, Pr, and inclination angle θ in the ranges, $1 \leqslant Pr \leqslant 25$ and $0 \leqslant \theta \leqslant \pi/2$. For large Pe values, the average rate of heat transfer, Nu, was found to behave closely to the theoretical result $Nu \approx Pe^{1/3}$, where $Pe = RePr$ is the Peclet number. D'Alessio discussed in [3] the two-dimensional problem of forced convection past an inclined elliptic cylinder. Both the steady state and unsteady state cases have been considered for moderate Reynolds numbers, $40 \leqslant Re \leqslant 70$ and Prandtl number $Pr = 1$. Badr [4] analysed numerically the laminar forced convection from a straight isothermal tube of elliptic cross-section placed in a uniform stream. For Reynolds number in the range 20–500, it is shown that: (a) the heat transfer reaches its maximum value when the angle of inclination is null while the minimum occurs when the angle of inclination is equal to $\pi/2$; (b) when the angle of inclination is equal to zero, smaller axis ratio gives higher heat transfer rates. Forced and mixed convective heat transfer from accelerated flow past an elliptic cylinder was investigated in [5]. The fluid is considered viscous and incompressible and the flow laminar and two-dimensional. The elliptic cylinder is inclined at an angle θ with the horizontal and starting from rest, accelerates uniformly through the fluid. The temperature of the cylinder

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Nomenclature

a	semi-major axis of elliptical cylinder
b	semi-minor axis of elliptical cylinder
c	focal distance
c_P	heat capacity
e	dimensionless focal distance or eccentricity, c/a
Pr	Prandtl number, $Pr = \mu/\alpha$
Re	Reynolds number, $Re = 2U_\infty a/\mu$
t	time
T	temperature
U_∞	free stream velocity
X	streamwise (horizontal) Cartesian coordinate
Y	transverse (vertical) Cartesian coordinate
Z	dimensionless temperature defined by the relations, $Z_{(c)} = \frac{T_{f(c)} - T_{f,\infty}}{T_{c,0} - T_{f,\infty}}$

Greek symbols

α	thermal diffusivity of the fluid phase
ε	axis ratio, b/a
μ	kinematic viscosity
ρ	density
τ	dimensionless time or Fourier number, $\tau = t\alpha/a^2$
ω	dimensionless vorticity
ψ	dimensionless stream function
Ξ	volume heat capacity ratio, $(\rho_c\ c_{P,c})/(\rho_f c_{P,f})$

Subscripts

c	refers to cylinder
f	refers to the fluid
0	initial conditions
∞	large distance from the cylinder

surface is constant. Khan et al. [6] used the Von Karman–Pohlhausen integral method to solve the boundary layer momentum and energy equations. Isothermal and isoflux thermal boundary conditions were considered on the surface of the cylinder. Three general correlations, one for drag and two for heat transfer, have been determined. The drag and the average heat transfer coefficients depend on the Reynolds number as well as on the axis ratio. It must be also mentioned that in [5] the experimental studies dedicated to fluid flow around and heat transfer from elliptical cylinders are reviewed.

In the articles mentioned previously the temperature of the cylinder is considered constant. A constant temperature inside the cylinder indicates the presence of a heat source in the system. When there is no heat source in the system, the heat transfer problem should be rewritten and solved as an unsteady conjugate heat transfer problem. The internal and external problems are the asymptotic formulations of the conjugate problem. The usefulness and at the same time the necessity of a study dedicated to the internal and external problems can be twofold argued. First, there are enough real life situations well described by these models. Secondly, when solving the conjugate problem, the asymptotic solutions play an important role.

The aim of this paper is to extend the previous studies to the case of elliptic cylinders with spatially uniform, but changing with time, temperature (i.e. to solve the external problem). The influence of the volume heat capacity ratio and axis ratio on the heat transfer rate is investigated for $Re = 5$, 10.0, 25.0, 40.0 (Re is the cylinder Reynolds number based on the major axis) and three values of the Prandtl number, $Pr = 1$, 10 and 100. From our knowledge, this problem was not investigated until now.

2. Model equations

Consider uniform flow of a Newtonian fluid past a fixed elliptic cylinder with major axis $2a$ and minor axis $2b$. The cylinder is oriented so that the major axis is parallel to the free stream flow direction. The flow is assumed to be laminar, steady and two-dimensional. The free stream velocity and temperature are denoted by U_∞ and $T_{f,\infty}$, respectively. The following statements are considered valid:

- the effects of buoyancy and viscous dissipation are negligible;
- the physical properties of the material of the cylinder and the fluid are considered to be uniform, isotropic and constant;
- no emission or absorption of radiant energy;
- no phase change.

The condition of spatially uniform temperature inside the cylinder is fulfilled if the relaxation time inside the cylinder is considerably smaller than the relaxation time in the fluid. The transfer is hundreds of times (at least) faster inside the cylinder than in the fluid. In terms of physical quantities, this condition means values considerably greater than one for the conductivity ratio (the conductivity ratio is defined as (cylinder's thermal conductivity)/(fluid thermal conductivity)).

The Cartesian coordinate system is not convenient for either analytical or numerical purposes. The conformal transformation

$$X + iY = \cosh(\xi + i\eta)$$

generates a coordinate system (elliptic cylindrical coordinates, [7]) that is better suited to the geometry of the prob-