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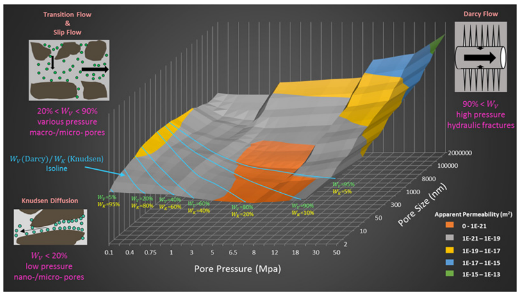
Full Length Article

Modeling of permeability for ultra-tight coal and shale matrix: A multi-mechanistic flow approach

Yi Wang[a,b], Shimin Liu[b,*], Yixin Zhao[a,c]

[a] College of Resources & Safety Engineering, China University of Mining and Technology, Beijing 100083, China
[b] Department of Energy and Mineral Engineering, G^3 Center and Energy Institute, The Pennsylvania State University, University Park, PA 16802, USA
[c] State Key Laboratory of Coal Resources and Safe Mining, China University of Mining and Technology, Beijing, China

GRAPHICAL ABSTRACT



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ABSTRACT

Gas transport in coal and shale matrices does not always fall into the continuum flow regime described by Darcy's law. Rather, a considerable portion of this transport is sporadic and irregular when the mean free path of gas becomes comparable to the prevailing pore scale. A nonlinear process influenced by non-Darcy flow components like gas sorption, gas slippage, and diffusion occurs throughout gas recovery. Therefore, a new permeability model with pressure-dependent weighting factors is presented to describe gas flow. This model contains the coupling of matrix flow with explaining the impact of both multiple flow regimes and stress-strain relationship on unconventional gas permeability evolution. The stress-strain relationships were derived from thermal-elastic equations and can be incorporated into the fracture-based flow component, enabling permeability prediction under uniaxial strain and hydrostatic conditions. The "U-shape" permeability trends caused by flow dynamics and geomechanical effects are observed in modeling results, which match experimental data. The agreement between modeling results and experimental data shows that gas permeability can be fully characterized by the presented model. This model has the ability to predict uniaxial strain permeability to hydrostatic permeability in a laboratory scale.

1. Introduction

Gas production from unconventional reservoirs, such as coal and shale, has become a major source of energy in the United States. It is well known that coal and shale are self-sourced reservoir rocks with pores ranging in size from nanometer to millimeter scale [1–3]. This multiscale pore structure accommodates various flow regimes resulting in complex gas transport behaviors. As expected, the gas transport process in shale reservoirs includes both Darcy and non-Darcy flow components [4–6], modeled as a weighted combination of Darcy and

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[*] Corresponding author.
E-mail address: szl3@psu.edu (S. Liu).

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Table 1
Nomenclature.

Symbol	Definition and Units
C_f	Cleat compressibility (Pa^{-1})
D_f	Fractal dimension of the pore surface (dimensionless)
d_m	Molecular diameter (m)
E	Young's modulus (Pa)
E_A	Modulus of the solid expansion (Pa)
G, λ	Lame constants of the porous rock (Pa)
L	Flow length (m)
J_K	Knudsen molar flux ($mol{\cdot}m^{-2}{\cdot}s^{-1}$)
J_T	Total mass flux ($mol{\cdot}m^{-2}{\cdot}s^{-1}$)
J_V	Viscous flow molar flux ($mol{\cdot}m^{-2}{\cdot}s^{-1}$)
k	Darcy permeability (m^2)
k_0	Initial Darcy permeability (m^2)
k_a	Total permeability (m^2)
k_{a0}	Initial total permeability (m^2)
k_B	Boltzmann constant (1.3805×10^{-23} J/K)
k_{K0}	Initial Knudsen permeability (m^2)
M	Molar mass (kg/mol)
p	Pore pressure (Pa)
p_0	Initial pore pressure (Pa)
P_L	Pressure at which coal attains half of the maximum adsorption amount (Pa)
P_ε	Pressure at which coal attains half of the maximum strain (Pa)
R	Universal gas constant ($J{\cdot}mol^{-1}{\cdot}K^{-1}$)
r	Pore radius (m)
T	Temperature (K)
t_m	Average time required for one collision between the molecules (s)
t_s	Average time required for one collision between nanopore wall and molecule (s)
t_t	Average time consumed for one collision of overall gas molecules (s)
V_L	Maximum adsorbed volume which can be achieved at infinite pressure (m^3/kg)
V_0	Gas molar volume (m^3/mol)
v_M	Average velocity of molecule movement (m/s)
W_K	Weighting coefficient of Knudsen diffusion (dimensionless)
W_V	Weighting coefficient of Knudsen diffusion (dimensionless)
Greek symbols	
α_T	Coefficient of volumetric thermal expansion (dimensionless)
Δp	Pressure drop across the flow length (Pa)
$\Delta\varepsilon$	Sorption-induced incremental volumetric strain (dimensionless)
δ'	Ratio of normalized molecular size d_m to local average pore diameter d_p (dimensionless)
δ_{ij}	Kronecker delta (dimensionless)
ε	Volumetric strain (dimensionless)
ε_{ij}	Linear strain (dimensionless)
ε_l	Maximum strain which can be achieved at infinite pressure (dimensionless)
ε_s	Macroscopic volumetric matrix shrinkage strain induced by gas desorption from coal (dimensionless)
ϕ	Porosity (dimensionless)
λ_g	Gas mean free path (m)
λ_T	Average free path of the overall gas molecules (m)
μ	Gas viscosity (Pa·s)
ρ	Gas density (kg/m^3)
ρ_s	Density of the solid adsorbent (kg/m^3)
σ	Hydrostatic stress (Pa)
σ_0	Initial hydrostatic stress (Pa)
τ	Tortuosity (dimensionless)
τ_{ij}	Total stress (Pa)
ν	Poisson's ratio (dimensionless)

Knudsen flows [7]. The breakdown of different flow types, such as Darcy flow, slip flow, transition flow, and Knudsen diffusion are classified and distinguished by the Knudsen number (K_n) which includes a function of pore size and pore pressure:

$$K_n = \frac{\lambda_g}{2r} \tag{1}$$

λ_g is gas mean free path defined as Table 1.:

$$\lambda_g = \frac{k_B T}{\sqrt{2}\pi d_m^2 p} \tag{2}$$

The gas flow regime transitions from continuum flow to Darcy flow ($K_n < 0.001$) and to Knudsen diffusion ($K_n > 10$), as shown in Fig. 1. This transition zone includes slip flow ($0.001 < K_n < 0.1$) and transition flow (0.1 < PermeabilitiesK_n < 10), in which neither continuum flow nor free molecular flow occur [8]. Previously proposed analytical models have involved a combination of transport mechanisms with weighting coefficients [7,9–17]. The literature is summarized in Table 2. The models listed in Table 2 are capable of describing gas transport under fixed-pore conditions. However, the dynamic pore structure changes because of changing stresses and structures that have not been considered in these models. In deep reservoir rocks, the depletion of pore fluids can significantly modify the state of stress in subsurface formations [18], and the induced stress variation can result in a dynamic change of the pore structure. For micropores and mesopores, a slight change of pore size can change the flow regime because the Knudsen number is sensitive to the pore radius for small pores. Specifically, effective stress changes can control pore structure and flow path length in coal and shale [8,19]. With depleting reservoir pressure, the increasing effective stress tends to close pre-existing fractures and reduces the permeability [20,21]. For coal, desorption-induced matrix shrinkage could offset the stress-induced permeability loss and even increase permeability with gas depletion [22,23]. Since permeability is determined by effective stress and matrix shrinkage, both effects should be considered when evaluating the evolution of permeability. Some research efforts have proposed to predict the evolution of permeability for elastic porous media under various boundary conditions, demonstrating that the evolution of permeability depends on the effective stress and flow-induced matrix deformation [7,20,21,24–28]. By combining the features of the hydrodynamic permeability models and stress-induced permeability models, it is possible to develop an analytical model to describe the evolution of permeability in coal and shale with gas depletion, CO_2 enhanced gas recovery, CO_2 sequestration, and so on.

In this study, a multi-mechanistic permeability model for coal and shale was developed under uniaxial strain (best-replicated *in situ* condition) and isotropic hydrostatic conditions. This new model incorporates pressure-dependent weighting coefficients to integrate the contributions of different gas transport mechanisms on permeability. Hydrodynamic flow and stress-induced fracture permeability models were coupled to represent Knudsen diffusive flow and Darcy flow. The transformation of this model under normal hydrostatic conditions was derived for different scenarios. A general discussion and an illustrative 3-D graph were presented to analyze our model.

2. Review of gas transport and permeability models

2.1. Hydrodynamic models of gas permeability in nanopores

The rate of gas flow between shale matrix and a fracture surface is very slow [4,7] and is considered to be of limited importance from the standpoint of gas production. Diffusion, however, plays an important role in matrix gas flow, and diffusivity is a critical property of tight reservoir rocks that can limit production where fracture permeability is several orders of magnitude higher than matrix permeability. Diffusion in coal and shale reservoirs is described by Fick's Law and is driven by the concentration gradient at which K_n is relatively high ($K_n > 10$). Fick's Law is commonly used to depict the relationship between diffusional influx and concentration gradient by assuming that the molar flow rate across a surface is proportional to the concentration gradient, the area of the surface, and the diffusion coefficient for a micro-porous medium. Diffusion in the shale matrix depends upon the matrix structure and concentration [4,7,29].

The Navier-Stokes equation that accounts for Darcy's law only works for the continuum flow regime, which includes Darcy viscous flow and slip flow. It breaks down when the flow regime moves into the transition zone and the free molecular flow zone [4,30]. In this case, the