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A scaling analysis to characterize thermomagnetic convection

Achintya Mukhopadhyay [a], Ranjan Ganguly [b,c], Swarnendu Sen [a], Ishwar K. Puri [d,*]

[a] Department of Mechanical Engineering, Jadavpur University, Kolkata 700 032, India
[b] Department of Mechanical and Industrial Engineering, University of Illinois at Chicago, Chicago, IL 60607, USA
[c] Department of Power Engineering, Jadavpur University, Kolkata 700 032, India
[d] Department of Engineering Science and Mechanics, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061, USA

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Abstract

Thermomagnetic convection is characterized using scaling arguments. We consider a square enclosure filled with a ferrofluid that is under the influence of an external magnetic field created by a line dipole. The height-averaged Nusselt number scales with the magnetic Rayleigh number as $Nu \sim Ra_{\mathrm{m}}^{0.25}$. This result is in excellent agreement with predictions obtained from detailed numerical simulations. Use of the Langevin equation of ferrofluid magnetization identifies an optimum enclosure height for which the Nusselt number reaches a maximum value for a given line dipole strength.

1. Introduction

Thermogravitational or free convection occurs under many circumstances and is used in numerous applications. However, its effectiveness greatly diminishes at small length scales as other effects become dominant. Buoyancy-induced convection ceases to be effective in reduced gravity, e.g., in spacecraft. Thermomagnetic convection is a feasible method to augment or suppress free convection for small length scale applications or in hypogravity [1,2], but it is not yet fully characterized. A thorough understanding of the relation between an applied magnetic field and the resulting heat transfer is necessary for the proper design and control of thermomagnetic devices.

Finlayson [1] first discussed thermomagnetic convection and provided a critical stability parameter beyond which this form of convection occurs. Schwab et al. [3] conducted an experimental investigation of the convective instability in a horizontal layer of ferrofluid and characterized the influence of the magnetic Rayleigh number on the Nusselt number. Krakov and Nikiforov [4] addressed the influence of the relative orientation of the temperature gradient and magnetic field on thermomagnetic convection in a square cavity. Yamaguchi et al. [5,6] performed experiments in a square enclosure and characterized the heat transfer in terms of a magnetic Rayleigh number.

However, all of these investigations assumed uniform magnetic fields, which, in most practical heat transfer applications, are not generally realizable. Moreover, the gradient associated with a nonuniform magnetic field is an important component of the thermomagnetic force. Therefore, while these previous studies are significant, they do not fully describe the influence of magnetic

* Corresponding author. Tel.: +1 540 231 3243; fax: +1 540 231 4574.
E-mail address: ikpuri@vt.edu (I.K. Puri).

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Nomenclature

B	magnetic field (T)
C_p	specific heat at constant pressure (kJ/kg K)
D	enclosure height (m)
h	heat transfer coefficient (W/m^2)
H	defined in Eq. (2) (A/m)
$\mathbf{J}_\mathrm{f}$	free current density (A/m^2)
k	thermal conductivity (W/mK)
k_B	Boltzmann constant (1.3807×10^{-23} J/K)
m	magnetic moment (strength) per unit length of line dipole (A m)
M	magnetization (A/m)
Nu	Nusselt number (hD/k) (dimensionless)
Pr	Prandtl number (ν/α) (dimensionless)
Ra_m	magnetic Rayleigh number (dimensionless)
t	time (s)
T	temperature (K)
u	x-component of velocity (m/s)
v	y-component of velocity (m/s)
V	velocity vector (m/s)
Greek symbols	
α	thermal diffusivity (m^2/s)
β_ρ	fluid compressibility ($-\frac{1}{\rho}\frac{\partial\rho}{\partial T}$) (K^{-1})
χ_0	magnetic susceptibility at reference temperature (dimensionless)
χ_m	magnetic susceptibility at operating temperature (dimensionless)
δ_T	thermal boundary layer thickness (m)
ϕ	coordinate direction
μ	viscosity (Pa s)
λ	Langevin parameter
μ_0	permeability of vacuum (1.257×10^{-6} N/A^2)
ν	kinematic viscosity (m^2/s)
ρ	density (kg/m^3)

field gradients for practically realizable thermomagnetic convection applications. Tangthieng et al. [7] provided a numerical analysis in the presence of a nonuniform field (such as one produced by a permanent magnet), but with two magnetic monopoles.

A few other researchers have considered spatially nonuniform magnetic fields during experimental or numerical investigations, but did not completely represent the variations in those fields [8–12]. In some cases, the field descriptions were inconsistent [7,13] since they were not in accord with the Maxwell's equations of electromagnetism. Consequently, it is not entirely appropriate to employ their correlations between the magnetic field attributes and the resulting heat transfer to design applications. Odenbach [14,15] performed elegant experiments to demonstrate the influence of the thermomagnetic destabilization force in microgravity using an azimuthal magnetic field with a radial gradient (such as one produced by a single current-carrying conductor). Although this magnetic field is realistic, the heat-transfer enhancement as a function of magnetizing current was not characterized in that investigation.

Recently, Ganguly et al. [2,16] addressed these issues by simulating free and forced thermomagnetic convection by considering a two-dimensional magnetic field that is similar to one created by a practical line-source dipole. They have shown that magnetic effects on the corresponding flow are localized. They found that while the addition of dipoles is beneficial for heat transfer, since they create additional recirculation zones, the enhancement in the overall heat transfer depends on the net magnetizing current alone. Wang and Wakayama [17] investigated natural convection with non-conducting and low-conducting diamagnetic fluids with magnetic fields that had different orientations. Tagawa et al. [18] and Kim and Hyun [19] investigated the interaction of thermogravitational and thermomagnetic convection in cubical enclosures under the influence of a magnetic field produced by a pair of electrical coils placed parallel to one pair of faces of the cavity.

Ganguly et al. [16] have shown that the thermomagnetic convection becomes more effective at small length scales, which makes this mode of heat transfer potentially attractive for MEMS applications. However, velocity and temperature measurements at the microscale are challenging. Therefore, some laboratory-scale experiments conducted at larger length scales will need to be appropriately scaled down to extrapolate experimental observations to actual microscale applications. Unfortunately, none of the above work adequately discusses scaling effects for thermomagnetic convection.

2. Formulation

Our configuration is similar to that of Ganguly et al. [16]. Fig. 1(a) presents schematic diagrams of the idealized configurations that we have investigated. The rectangular cavity in Fig. 1(a) extends to infinity (i.e., it has a large depth) in the third dimension such that the flow that develops inside it is two-dimensional. The left hand side vertical wall is maintained at a temperature of T_h while the other vertical wall is an isothermal heat sink at T_c. The upper and lower walls are adiabatic. A line dipole, which provides the external magnetic field, is placed adjacent to the lower wall halfway along the