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Large-eddy simulation of combined forced and natural convection in a vertical plane channel

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Abstract

A combined forced and natural convective flow between two vertical plates with different temperatures is studied using large eddy simulation. The numerical simulations were performed with a Grashof number of $Gr = 9.6 \times 10^5$ and Reynolds number of $Re_\tau = 150$ (based on the wall friction velocity and half channel width). Two sets of dynamic subgrid-scale (SGS) models were tested in the simulation; namely, the set of linear SGS models consisting of the dynamic Smagorinsky SGS stress model (DM) and dynamic eddy diffusivity SGS heat flux model (DEDM-HF), and the set of nonlinear SGS models consisting of the dynamic nonlinear SGS stress model (DNM) and dynamic tensor diffusivity SGS heat flux model (DTDM-HF). The numerical results are compared with the reported direct numerical simulation data. It is found that the resolved and SGS quantities related to the temperature field are noticeably influenced by the choice of SGS models. In general, the set of dynamic nonlinear SGS models yields better prediction of the flow than the set of dynamic linear SGS models.

Keywords: Turbulence; Convection; Buoyancy; Heat transfer; Large-eddy simulation; Subgrid-scale model

1. Introduction

In mechanical and environmental engineering, combined (mixed) forced and natural turbulent convection is a frequently encountered thermal-fluid phenomenon, which exists, for example, in the atmospheric environment, urban canopy flows, ocean currents, gas turbines, heat exchangers, nuclear reactors, and computer chip cooling systems. In the early development of the subject of convective heat transfer, forced and natural convections were studied separately and the interaction between these two physical processes was ignored. Modern research on combined forced and natural convection was initiated in the 1960's based on experimental approaches (see Metais and Eckert [1]). Since then, refined experimental measurements have become available [2,3], and the research methodology has been extended to numerical simulations based on the Reynolds averaged Navier–Stokes (RANS) method [4,5] and direct numerical simulations (DNS) [6]. A detailed review of the subject (up to 1989) can be found in Jackson et al. [7]. Recently, large-eddy simulation (LES) has been utilized for studying this type of flow, which includes the work of Lee et al. [8] who studied a heated vertical annular pipe flow, that of Zhang and Chen [9] who studied indoor air flows, that of Yan [10] who investigated thermal plumes for different initial conditions, that of Tyagi and Acharya [11] who studied heat transfer in a rotating channel flow with rib turbulators, and that of Wang et al. [12] who studied combined forced and natural convective channel flow using a general dynamic linear tensor thermal diffusivity (or simply, tensor diffusivity) subgrid-scale heat flux model.

By their nature, buoyancy driven turbulent flows are unsteady and feature both large and fine scale flow

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Nomenclature

a_j, b_j	base vectors
C_f	resolved friction coefficient: $2\tau_w/\rho U_D^2$
c_P	specific heat at constant pressure
C_S, C_W, C_N	SGS stress model coefficients
C_θ^T	SGS heat flux model coefficient
D	distance from the wall to the maximum streamwise velocity location
D_{ij}	tensor thermal diffusivity
g_i	gravitational acceleration vector: $[-g, 0, 0]^T$
Gr	Grashof number: $g\beta\Delta\theta(2\delta)^3/\nu^2$
h_j	grid level SGS heat flux vector
H_j	test-grid level SGS heat flux vector
$\mathscr{L}_{ij}$	Leonard type stress tensor
$\mathscr{L}_j$	vector
M_{ij}, W_{ij}, N_{ij}	differential tensors
M_j	differential vector
Nu	Nusselt number: $2q_w/[\lambda(\theta_D - \theta_w)/D]$
p	pressure
q_w	wall heat flux
Re_τ	Reynolds number based on friction velocity: $u_\tau\delta/\nu$
Re_b	Reynolds number based on bulk velocity: $U_b\delta/\nu$
$\overline{S}_{ij}$	resolved strain rate tensor: $(\partial\bar{u}_i/\partial x_j + \partial\bar{u}_j/\partial x_i)/2$
T_τ	wall friction temperature: $q_w/(\rho c_P u_\tau)$
T_{ij}	test-grid level SGS stress tensor
U_b	bulk velocity across the channel: $\int_0^{2\delta}\langle\bar{u}_1\rangle\,dx_2/(2\delta)$
U_D	bulk velocity averaged over the distance D: $\int_0^D\langle\bar{u}_1\rangle\,dx_2/D$
u_i	velocity components: $i = 1, 2, 3$
u_τ	wall friction velocity: $\sqrt{\tau_w/\rho}$
α	molecular thermal diffusivity: $\lambda/(\rho c_P)$
α_{sgs}	SGS eddy thermal diffusivity
α_{ij}, γ_{ij}, η_{ij}	grid level tensors
β	thermal expansion coefficient
β_{ij}, λ_{ij}, ζ_{ij}	test-grid level tensors
δ	half channel width
δ_{ij}	Kronecker delta
Δ	mesh or filter length
Γ	effective eddy diffusivity for turbulent heat flux
λ	thermal conductivity
ν	kinematic viscosity
ν_{sgs}	SGS eddy viscosity
$\overline{\Omega}_{ij}$	resolved rotation rate tensor: $(\partial\bar{u}_i/\partial x_j - \partial\bar{u}_j/\partial x_i)/2$
ρ	density
σ_{sgs}	SGS Prandtl number
θ	temperature
θ_b	bulk temperature across the channel: $\int_0^{2\delta}\langle\bar{\theta}\rangle\,dx_2/(2\delta)$
θ_D	bulk temperature averaged over the distance $D: \int_0^D\langle\bar{\theta}\rangle\,dx_2/D$
Θ_r	reference temperature
τ_w	wall shear stress
τ_{ij}	grid level SGS stress tensor
ϕ	a function of space

Subscripts and Superscripts

$(\cdot)_1$	streamwise component
$(\cdot)_2$	wall-normal component
$(\cdot)_3$	spanwise component
$(\cdot)^a$	averaged value over two walls
$(\cdot)_c$	value at cold wall
$(\cdot)_h$	value at hot wall
$(\cdot)_i$, $(\cdot)_j$, $(\cdot)_{ij}$	vectors or tensors: $i, j = 1, 2, 3$
$(\cdot)_{ij}^*$	a trace-free tensor: $(\cdot)_{ij}^* = (\cdot)_{ij} - (\cdot)_{kk}\delta_{ij}/3$
$(\cdot)_{rms}$	root-mean-square
$(\cdot)_{sgs}$	subgrid-scale component
$(\cdot)_w$	value at the wall
$\overline{(\cdot)}$	grid level filter
$\widetilde{(\cdot)}$	test-grid level filter
$(\cdot)^+$	wall coordinates
$(\cdot)''$	residual component relative to a time- and plane-averaged quantity
$\langle\cdot\rangle$	time- and plane-averaged quantity

structures. Therefore, in comparison with the RANS method which is based on the concept of ensemble averages, a time dependent and fine scale resolved calculation based on DNS or LES can provide more details of the temperature and fluid flow fields. In LES of buoyant flows, the unknown subgrid-scale (SGS) stress tensor and heat flux (HF) vector associated with the unresolved scales of motion need to be modelled to close the set of governing equations. Although in a direct sense, the SGS stress model is for closing the filtered momentum equation and the SGS HF model is for closing the filtered thermal energy equation, these two types of SGS models jointly influence both the velocity and temperature fields. This is because the transport of momentum is tightly coupled with that of thermal energy in a combined forced and natural convective flow, temporally and spatially. In the following context, we briefly review the SGS stress and HF modelling approaches that are relevant to this research.

The conventional Smagorinsky type Dynamic model (DM) introduced by Germano et al. [13] and Lilly [14] is known for its capability of self-calibration, general balancing of the turbulence kinetic energy (TKE) between the resolved and unresolved scales, and being free from any empirical constants and artificial near-wall damping functions. However, the DM originates from the Smagorinsky constitutive relation which is based on the molecular transport analogy and requires the principal axes of the negative SGS stress tensor to be strictly aligned with those of the