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A fractal resistance model for flow through porous media

Jinsui Wu [a,b], Boming Yu [a,*]

[a] *Department of Physics, Huazhong University of Science and Technology, 1037 Luoyu Road, Wuhan 430074, PR China*
[b] *Department of Physics, North China Institute of Science and Technology, Yanjiao Town, Sanhe City, Hebei 06520, PR China*

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Abstract

A fractal model for resistance of flow through porous media is developed based on the fractal characters of porous media and on the pore–throat model for capillary. The proposed model is expressed as a function of the pore–throat ratio, porosity, property of fluid, pore/capillary and particle sizes, fluid velocity (or Reynolds number) and fractal dimensions of porous media. There is no empirical constant and every parameter has clear physical meaning in the proposed model. The model predictions are compared with experiment data, and good agreement is found between them.

Keywords: Porous media; Fractal; Flow resistance; Laminar flow

1. Introduction

The widely employed resistance model for flow through porous media was proposed by Ergun [1] in 1952. This model is called Ergun equation:

$$\frac{\Delta P}{L_0}=\frac{150\mu(1-\varepsilon)^2 v_s}{D_p^2\varepsilon^3}+1.75\frac{1-\varepsilon}{\varepsilon^3}\frac{\rho v_s^2}{D_p} \tag{1}$$

where ΔP is the pressure drop, L_0 is the length along the macroscopic pressure gradient in porous media, v_s is the superficial velocity (defined by $v_s = Q/A$, where Q is the total flow rate through a cross section of area A), μ is the absolute viscosity of fluid, ε is the porosity, and D_p is the appropriate characteristic length for a medium or the equivalent mean diameter of particles, ρ is the density of fluid. Eq. (1) is based on the average hydraulic radius [2]. The first term on the right side of Eq. (1) is called Blake-Kozeny equation, i.e. $\frac{\Delta P}{L_0}=\frac{150\mu(1-\varepsilon)^2 v_s}{D_p^2\varepsilon^3}$, which represents the viscous energy loss primarily in laminar flow, and pressure drop for flow at low speed (or low Reynolds number) is mainly determined by the viscous energy loss, i.e. when the modified Reynolds number ($Re_p = (D_p\rho v_s/\mu)(1-\varepsilon)^{-1}$) is less than 10 [2]. If the first term on the right side of Eq. (1) is neglected, Eq. (1) is reduced to $\frac{\Delta P}{L_0}=1.75\frac{1-\varepsilon}{\varepsilon^3}\frac{\rho v_s^2}{D_p}$, which is called Burke-Plummer equation. Burke-Plummer equation denotes the kinetic energy loss primarily in turbulent flow and the kinetic/local energy loss dominates the pressure drop when the modified Reynolds number Re_p is higher than 100 [2]. The interior mechanism for the kinetic energy loss is not well understood. Ergun equation indicates that the pressure drop across the packing length is dependent upon the flow rate, the viscosity and density of fluid, and the size, shape and surface of packing materials [1]. It has been shown that the pressure loss as indicated by Eq. (1) is obtained by adding the viscous and kinetic energy losses. Ergun equation has been hotly debated in the past decades. Many investigators [3–10] discussed its applicability and different empirical constants under different porosities and particles.

It has been shown that the fractal geometry theory [11] has been used as a tool in many disciplines to characterize irregular or disordered objects [11–13] such as coast lines, clouds and islands, roughness of surfaces [14–16], sandstone pores [17,18], fracture surfaces of metal [19], and granular materials [20], etc. The pores and their distributions in porous media are analogous to islands or lakes

* Corresponding author. Tel.: +86 27 87542153; fax: +86 27 87543882.
E-mail address: yuboming2003@yahoo.com.cn (B. Yu).

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on earth and to contact spots on engineering surfaces. Therefore, it is possible to model the transport properties such as flow resistance and permeability for flow in porous media by fractal geometry theory. In the light of this point, Yu et al. [21,22] proposed a fractal geometry model for permeability of porous media and their model has been shown to be suitable not only for particle porous media [21] but also for porous fabrics [22]. Their model is analytically expressed as a function of fractal dimensions (for pore spaces and for tortuous capillaries/streamlines) and microstructural parameters of the media. Karacan and Halleck [23] extended Yu and Cheng's [21] model to the prediction of the permeability for grain fragments. Recently, Shi et al. [24,25] extended Yu et al.'s fractal permeability model [21,22] to modeling the permeability for the gas diffusion layer (GDL) of PEM fuel cells, whose pore size is in the order of 10^{-5}–10^{-8} m. Meng et al. [26] also applied the fractal geometry theory to model the permeation of membrane fouling in membrane bioreactor. The cake layer formed on membrane surface presents a major challenge to membrane permeation, and it can be considered as a porous media. The cake layer permeability was derived and found to be a function of the pore-area fractal dimension and microstructural parameters.

From the above brief review, it is seen that the wide applications of the fractal geometry theory in many fields have been found. It, therefore, may be possible to develop the analytical model for resistance of flow in porous media based on the fractal geometry theory. In this paper, we derive a fractal model for resistance of flow through porous media with particles of different shapes based on the fractal characters of the media and on the pore–throat model for capillary. In the following section, the fractal characters of porous media are addressed first.

2. Fractal characters of porous media

It has been shown that the cumulative size distribution of pores in porous media follows the fractal scaling law [21,22]:

$$N(L \geqslant \lambda) = (\lambda_{\max}/\lambda)^{D_f} \tag{2}$$

where λ is the diameter of pores, $\lambda_{\max}$ is the maximum diameter of pores, N is the cumulative population of pores whose sizes are greater than or equal to λ, and D_f is the fractal dimension for pores, with $1 < D_f < 2$ in two dimensions and $2 < D_f < 3$ in three dimensions.

It is evident that the total number of pores, from the smallest diameter to the largest diameter, can be obtained from Eq. (2) as

$$N_t(L \geqslant \lambda_{\min}) = (\lambda_{\max}/\lambda_{\min})^{D_f} \tag{3}$$

Differentiating on both sides of Eq. (2) results in

$$-\mathrm{d}N = D_f \lambda_{\max}^{D_f} \lambda^{-(D_f+1)}\,\mathrm{d}\lambda \tag{4}$$

Dividing Eq. (4) by (3) yields

$$-\frac{\mathrm{d}N}{N_t} = D_f \lambda_{\min}^{D_f} \lambda^{-(D_f+1)}\,\mathrm{d}\lambda = f(\lambda)\,\mathrm{d}\lambda \tag{5}$$

where $f(\lambda) = D_f \lambda_{\min}^{D_f} \lambda^{-(D_f+1)}$ is the probability density function. The probability density function $f(\lambda)$ should satisfy the following normalization relation:

$$\int_{\lambda_{\min}}^{\lambda_{\max}} f(\lambda)\,\mathrm{d}\lambda = 1 - \left(\frac{\lambda_{\min}}{\lambda_{\max}}\right)^{D_f} = 1 \tag{6}$$

As a result, Eq. (6) holds if and only if [27]

$$(\lambda_{\min}/\lambda_{\max})^{D_f} = 0 \tag{7}$$

In general, $\lambda_{\min}/\lambda_{\max} \leqslant 10^{-2}$ in porous media, and Eq. (7) holds approximately, thus the fractal geometry theory and technique can be used to analyze properties of porous media. In above equations, fractal dimension D_f is given by [27]

$$D_f = d_E - \frac{\ln \varepsilon}{\ln(\lambda_{\min}/\lambda_{\max})} \tag{8}$$

where d_E is the Euclidean dimension, and $d_E = 2$ (3) in two (three) dimensions.

If one is interested in fractal particles, the above parameters and equations are immediately applicable as long as appropriate changes are made, for example, changing the porosity ε into the particle volume fraction, and pore diameter λ into the particle diameter, etc.

The tortuous capillaries have also been shown to follow the fractal scaling law given by [21]

$$L_t(\lambda) = \lambda^{1-D_T} L_0^{D_T} \tag{9}$$

where D_T is the fractal dimension for tortuous capillaries with $1 < D_T < 2$ in two dimensions, representing the convoluted extent of capillary pathways for fluid flow through a porous medium, and $L_t(\lambda)$ is the tortuous/real length. Due to the tortuous nature of the pore channel, $L_t(\lambda) \geqslant L_0$, where L_0 is the length along the macroscopic pressure gradient in the medium. Note that $D_T = 1$ represents a straight capillary path, and a higher value of D_T corresponds to a highly tortuous capillary.

Based on the above fractal characters of pores and tortuous capillaries in porous media, a fractal model for resistance of flow in porous media is derived in the following section.

3. Fractal model for flow resistances

3.1. The viscous energy loss along the flow path at low Reynolds numbers

In this model, we assume that a porous medium is comprised of a bundle of tortuous capillaries. The flow rate through a tortuous capillary is given by modifying the well known Hagen–Poiseulle equation as [28]

$$q(\lambda) = \frac{\pi}{128} \frac{\Delta P_1}{L_t} \frac{\lambda^4}{\mu} \tag{10}$$