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Viscoelastic fluid flow and heat transfer over a stretching sheet under the effects of a non-uniform heat source, viscous dissipation and thermal radiation

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Abstract

The problem of flow and heat transfer of an incompressible homogeneous second grade fluid over a non-isothermal stretching sheet in the presence of non-uniform internal heat generation/absorption is investigated. The governing partial differential equations are converted into ordinary differential equations by a similarity transformation. The effects of viscous dissipation, work due to deformation, internal heat generation/absorption and thermal radiation are considered in the energy equation and the variations of dimensionless surface temperature as well as the heat transfer characteristics with various values of non-dimensional viscoelastic parameter k_1, Prandtl number σ, Eckert number $E_c(E'_c)$, radiation parameter N_R, and the coefficients of space-dependent (A^*) and temperature-dependent (B^*) internal heat generation/absorption are graphed and tabulated. Two cases are studied, namely, (i) the sheet with prescribed surface temperature (PST case) and (ii) the sheet with prescribed heat flux (PHF case).

Keywords: Flow and heat transfer; Viscoelastic fluid; Frictional heating; Non-uniform heat source; Elastic deformation; Radiation; Stretching sheet

1. Introduction

Boundary layer behaviour over a moving continuous solid surface is an important type of flow occurring in a number of engineering processes. To be more specific, heat-treated materials travelling between a feed roll and a wind-up roll, aerodynamic extrusion of plastic sheets, glass fiber and paper production, cooling of an infinite metallic plate in a cooling path, manufacturing of polymeric sheets are examples for practical applications of continuous moving flat surfaces. Since the pioneering work of Sakiadis [1], various aspects of the problem have been investigated by many authors. Mass transfer's analyses at the stretched sheet were enclosed in their studies by Erickson et al. [2] and relevant experimental results were reported by Tsou et al. [3] regarding several aspects for the flow and heat transfer boundary layer problems in a continuously moving sheet. Crane [4] and Gupta and Gupta [5] have analyzed the stretching problem with constant surface temperature while Soundalgekar [6] investigated the Stokes problem for a viscoelastic fluid. This flow was examined by Siddappa and Khapate [7] for a special class of non-Newtonian fluids known as second-order fluids which are viscoelastic in nature.

Rajagopal et al. [8] independently examined the same flow as in Ref. [7] and obtained similarity solutions of the boundary layer equations numerically for the case of small viscoelastic parameter k_1. It is shown that skin-friction decreases with increase in k_1. Dandapat and Gupta [9] examined the same problem with heat transfer. In Ref. [9], an exact analytical solution of the non-linear equation governing this self-similar flow which is consistent with the

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Nomenclature

A, D	prescribed constants
A_0	constant (Eqs. (23) and (29))
A^*	space-dependent internal heat generation/absorption
B^*	temperature-dependent internal heat generation/absorption
c	stretching rate
c_P	specific heat at constant pressure
E_c	Eckert number for the PST case
E'_c	scaled Eckert number for the PHF case
f	dimensionless stream function
g	dimensionless temperature for the PHF case
k	thermal conductivity
k_1	viscoelastic parameter
k_0	parameter related to N_R
k^*	mean absorption coefficient
l	characteristic length
N_R	radiation parameter
q_r	radiative heat flux
q'''	rate of internal heat generation or absorption
r	parameter related to k_1 (Eq. (13))
T	fluid temperature
u, v	velocity components along x and y directions, respectively
x, y	Cartesian coordinates along the plate and normal to it, respectively

Greek symbols

α	thermal diffusivity
α_1, α_2	normal stress moduli
η	dimensionless similarity variable
θ	dimensionless temperature for the PST case
μ	dynamic viscosity
υ	kinematic viscosity
ρ	density
σ	Prandtl number
σ^*	Stefan–Boltzmann constant

Subscripts

w, ∞	conditions at the surface and in the free stream, respectively

Superscript

$(')$	derivative with respect to η

numerical results in Ref. [8] is given and the solutions for the temperature for various values of k_1 are presented. Later, Cortell [10] extended the work of Dandapat and Gupta [9] to study the heat transfer in an incompressible second-order fluid caused by a stretching sheet with a view to examining the influence of the viscoelastic parameter on that flow. It is found that temperature distribution depends on k_1, in accordance with the results in Ref. [9].

In the case of fluids of differential type (see Ref. [11]), the equations of motion are in general one order higher than the Navier–Stokes equations and, in general, need additional boundary conditions to determine the solution completely. These important issues were studied in detail by Rajagopal [11,12] and Rajagopal and Gupta [13].

The effects of heat generation/absorption become important in view of various physical problems (see Vajravelu and Hadjinicolaou [14]) and those effects have been assumed to be constant, space-dependent or temperature-dependent (Vajravelu and Hadjinicolaou [15]). Even, very recently, the mixed convection boundary layer flow of a Newtonian, electrically conducting fluid over an inclined continuously stretching sheet with power–law temperature variation in the presence of magnetic field, internal heat generation/absorption and wall suction/injection is analyzed by Abo-Eldahab and El Aziz [16]. In the present research, we extend the problem investigated in Ref. [16] to viscoelastic fluid flows.

Furthermore, Char [17] studied MHD flow of a viscoelastic fluid over a stretching sheet, however, only the thermal diffusion is considered in the energy equation; later, Sarma and Rao [18], Vajravelu and Roper [19] and Cortell [20,21] analyzed the effects of work due to deformation in such an equation. Another effect which bears great importance on heat transfer is the viscous dissipation. The determination of the temperature distribution when the internal friction is not negligible is of utmost significance in different industrial fields, such as chemical and food processing, oil exploitation and bio-engineering. Consequently, the effects of viscous dissipation are also included in the energy equation.

On the other hand, the effect of radiation on viscoelastic boundary-layer flow and heat transfer problems can be quite significant at high operating temperature. In view of this, viscoelastic flow and heat transfer over a flat plate with constant suction, thermal radiation and without viscous dissipation were studied by Raptis and Perdikis [22]. Viscous dissipation and radiation were considered by Raptis [23] and the effect of radiation was also included in Ref. [24] and in Ref. [25]. Very recently, researches in these fields have been conducted by many investigators [26–30]; however, the effects of work due to deformation on viscoelastic flows and heat transfer in the presence of radiation, viscous dissipation and non-uniform heat source/sink have not been studied in recent years.

In the present paper a proper sign for the normal stress modulus (i.e., $\alpha_1 \geqslant 0$) is used and, as we will see in Section 3, the effects of viscous dissipation, work due to deformation, internal heat generation/absorption and thermal radiation are included in the energy equation. This last effect has been enclosed in this study by employing the Rosseland