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Influence of capillary forces on water injection into hot rock, saturated with superheated vapour

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Abstract

The results of a theoretical study and numerical analysis of the role of capillary pressure of cold water injection into depleted geothermal reservoirs are presented. A simplified 1-D mathematical model is developed, that describes the motion of a sharp vaporization front. Some asymptotic estimates for a wide range of parameters are given and a similarity solution is derived. Analytical results are then compared with those obtained from the numerical reservoir simulator TOUGH2, showing a good agreement between the two.

Keywords: Water injection; Capillary pressure; Analytical solution; Numerical simulation

1. Introduction

The influence of capillary forces on fluid flow in porous media has been the subject of numerous theoretical and experimental studies over the past decade. The aim of the present study was to develop a mathematical model for a better understanding of the physical processes occurring in geothermal systems. One of the most important processes in geothermal reservoirs under exploitation and/or recharge is capillarity, as it has a substantial influence on fluid phase changes. The effects of capillarity on phase changes are also of significance to Hot Dry Rock technology. Capillary pressure effects in porous rocks were studied, among others, by Udell [1], Pruess and O'Sullivan [2], Pruess [3] and Li and Horne [4,5]. Pruess and O'Sullivan [2] reported the numerical simulations that were performed to evaluate the impact of capillarity and vapour adsorption on the depletion of vapour-dominated geothermal reservoirs.

The numerical simulator TOUGH2 [6] for multiphase heat and fluid flow represents a powerful tool for modelling a large part of the wide spectrum of physical phenomena occurring in geothermal reservoirs. However the treatment of simpler physical situations, for which solutions exist in a closed analytical form, is a necessary step towards our understanding of the essential features of the phenomena involved. Analytical solutions can, moreover, also prove the behavior and accuracy of complex numerical simulators.

Frontal solutions play a fundamental role in the theoretical investigation of phase transition problems. These solutions imply that phase transition takes place over a narrow region or sharp front that can be considered as a discontinuity of the water saturation function, and that this interface separates single-phase zones. This method has been applied to geothermal reservoir modelling, as in, for example, Udell [1], Pruess et al. [7], Garg and Pritchett [8], Woods and Fitzgerald [9], Barmin and Tsypkin [10], Woods [11] and Tsypkin and Woods [12]. As a rule, the frontal formulation admits the derivation of an explicit solution in some specific cases, and numerical methods are usually verified by comparing their results with analytical solutions. Traditionally, derivation of the exact and explicit solution is the first step in a new mathematical model, providing insight into physical processes. Recently,

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Nomenclature

a	thermal diffusivity [m^2/s]
C	specific heat [J/(K kg)]
C_p	specific heat of vapour at constant pressure [J/(K kg)]
d	typical length scale of a pore [m]
k	permeability [m^2]
L	length scale [m]
P	pressure [Pa]
P_c	capillary pressure [Pa]
P_f	pressure at the flat surface [Pa]
q	specific heat of vaporization [J/kg]
R	gas constant [J/(kg K)]
R_0	universal gas constant [J/(mol K)]
r	mean radius of the capillary meniscus [m]
t	time [s]
T	temperature [K]
V	velocity of the vaporization front [m/s]
V_m	molar volume of water [m^3/mol]
$\mathbf{v}$	filter velocity [m/s]
x	coordinate [m]
$X(t)$	position of the vaporization front [m]
Greek symbols	
α_w	water compressibility coefficient [1/Pa]
Δ	Laplace operator
γ	dimensionless similarity coordinate of the vaporization front
κ	conductivity coefficient [m^2/s]
θ	contact angle [degree]
λ	thermal conductivity [W/(m K)]
μ	viscosity [Pa s]
ρ	density [kg/m^3]
σ	surface tension [J/m^2]
ϕ	porosity
ζ	dimensionless similarity variable
Subscripts	
n	normal
0	initial value
s	porous medium skeleton
v	vapour
w	water
+	quantities to the right of the front
−	quantities to the left of the front
*	values of the quantities at the front
1	vapour domain
2	water domain
Superscript	
0	boundary value

we applied this analytical approach to the problem of vapour extraction from water-saturated geothermal reservoirs [13,14].

In the present paper we extend our analysis to the problem of cold water injection into hot rock saturated with superheated vapour, assuming that a sharp vaporization front separates the water- and vapour-saturated zones. This is a typical case in which the effects of capillarity on heat and mass transfer processes in porous rocks play a very important role.

The paper is organized as follows. In Section 2, a mathematical model of cold water injection into vapour-saturated hot rock is developed. The injection process leads to formation of the sharp liquid front that separates the water-saturated and vapour-saturated regions. A full system of boundary conditions at the interface, which takes into account capillary forces, is then derived. In Section 3, the problem is reduced to a system of transcendental equations by a similarity solution approach. Using asymptotic estimates and numerical modelling with TOUGH2, the influence of capillary forces on the boiling process induced by cold water injection is investigated. In Section 4 we compare the results of the analytical model and numerical simulations, confirming that two different boiling regime exist: (1) with formation of a two-phase transition zone and (2) with formation of a sharp vaporization front. The conclusions are given in Section 5.

2. Problem formulation

Consider the injection of pure cold water into a high-temperature geothermal reservoir saturated with superheated vapour. If the porous rock is initially superheated then, as the injected liquid vaporizes, a boiling front develops, producing vapour ahead of the front. In order to describe the dynamics and thermodynamics of liquid and vapour flow through the porous rock we use Darcy's law, mass and energy conservation laws, and the equation of state for water and vapour (see, for example, [15,16]). Being the movement of water and vapour through porous media usually slow, fluid and rock are assumed to be in local thermodynamic equilibrium, owing to thermal diffusion between the solid and fluid [16].

2.1. Basic equations

In the vapour region, combining the equations for vapour flow according to the above assumptions and laws, the following system of two equations for the temperature and pressure is obtained [14]:

$$\frac{\partial P_v}{\partial t} - \frac{P_v}{T}\frac{\partial T}{\partial t} - \frac{k}{\phi\mu_v}(\operatorname{grad} P_v)^2 = -\frac{k}{\phi\mu_v}\frac{P_v}{T}\operatorname{grad} P_v \operatorname{grad} T + \frac{k}{\phi\mu_v}P_v\Delta P_v \qquad (1)$$