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On the existence of parallel flow for mixed convection in an inclined duct

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Abstract

The necessary condition for the occurrence of parallel mixed convection flow in an inclined duct is determined by employing the Boussinesq approximation. A sample case involving an inclined infinitely-wide plane channel is discussed to illustrate this condition. It is shown that, according to the necessary condition, parallel flow cannot occur in this case. Indeed, the investigated flow is the superposition of a parallel streamwise flow and a secondary flow. An exponential equation of state for the fluid is assumed and the balance equations are solved analytically to determine the dimensionless velocity distribution, as well as the conditions for the occurrence of flow reversal.

Keywords: Laminar flow; Mixed convection; Boussinesq approximation; Inclined duct; Analytical methods

1. Introduction

Several theoretical and experimental investigations of laminar buoyancy-induced flows in vertical or inclined ducts are available in the literature. Earlier theoretical papers [1–3] are based on analytical solutions of the balance equations and point out the basic features of laminar mixed-convection flows in the fully developed regime of vertical ducts. These papers refer to the simplest cross-sectional shapes, i.e. plane-parallel channels, circular tubes and rectangular ducts. In Ref. [4], an interesting extension of the solution found in Ref. [1] for a plane-parallel vertical channel with isothermal walls having unequal temperatures is obtained. The authors release the Boussinesq approximation invoked in Ref. [1] and assume that the fluid properties change with temperature according to an ideal gas model.

In the last fifteen years, the analysis of mixed convection flows in vertical and inclined ducts has been the subject of several papers, mainly following the interest of these flows for engineering problems such as the cooling of electronic equipments and the design of solar collectors. The investigations presented in Refs. [5–9] are devoted to the analysis of either developing or fully developed flows, and cases such that flow reversal occurs are considered. In Ref. [7], an analytical solution based on Fourier series expansions is presented which yields the velocity and temperature field for fully-developed mixed convection in a vertical rectangular duct with a hotter isothermal wall and three cooler isothermal walls. In Ref. [8], a numerical solution of the balance equations is obtained for buoyancy-induced heat and mass transfer

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Nomenclature

A, B	quantities defined in Eq. (13)
b	distance between the channel walls
c_v	specific heat at constant volume
D	$=2b$, hydraulic diameter
$F(Y)$	dimensionless function defined by Eq. (32)
$\mathbf{g}$	gravitational acceleration
Gr	Grashof number, defined in Eq. (31)
k	thermal conductivity
p	pressure
P	difference between the pressure and the hydrostatic pressure
$\mathbf{r}$	position vector
Ra	Rayleigh number
Re	Reynolds number, defined in Eq. (31)
T	temperature
T_0	reference temperature
u, v	dimensionless velocity components defined in Eq. (31)
$\mathbf{U}$	fluid velocity
U_0	mean fluid velocity in a channel section
x, y, z	rectangular coordinates
Y	dimensionless coordinate defined in Eq. (31)

Greek symbols

β	volumetric coefficient of thermal expansion
Γ	ratio between Gr and Re
Γ_{fr}, $\widetilde{\Gamma}_{fr}$	threshold values of Γ for the onset of flow reversal, given by Eqs. (41) and (42)
$\Delta\varrho(T)$	difference between the mass density and the reference mass density
Λ	dimensionless parameter defined by Eq. (31)
μ	dynamic viscosity
Ξ	dimensionless parameter defined by Eq. (31)
ϱ	mass density
ϱ_0	reference mass density, i.e. mass density for $T = T_0$
φ	tilt angle defined by Eq. (21)
Φ	viscous dissipation function defined by Eq. (4)
$\psi(Y)$	local bending angle defined by Eq. (43)

Superscript

$'$	projection of a vector on the xy-plane

in a vertical rectangular duct such that three walls are adiabatic, while the fourth is kept at a uniform temperature or at a uniform heat flux. In Ref. [9], the laminar and parallel buoyancy-induced flow in a vertical rectangular duct is considered, providing also a theorem on the uniqueness of the parallel flow solution in vertical ducts of arbitrary shape.

The effect of viscous dissipation for parallel mixed convection flows is analysed either in a vertical plane channel [10] or in a vertical circular duct [11]; both solutions are obtained utilising a perturbation method. A perturbation series solution [12] refers to the case of combined forced and free flow with viscous dissipation in an inclined plane channel with isothermal walls having unequal temperatures. In this paper, it is shown that the tilt angle, the viscous dissipation effect and the buoyancy effect influence the distribution of the difference between the pressure and the hydrostatic pressure in a channel cross-section: this distribution is uniform for a vertical channel while it becomes nonuniform when the channel is inclined.

The main aim of the present paper is to state and prove a theorem defining the necessary condition for the occurrence of fully-developed parallel flow in an inclined duct with an arbitrary cross section. This theorem holds under the assumption of validity of the Boussinesq approximation as well as under the hypothesis that the thermal boundary conditions do not produce a net fluid heating in the axial direction. In order to illustrate the importance of this theorem, an example is discussed. The example refers to an inclined plane channel not fulfilling the necessary condition for the occurrence of parallel flow. Indeed, the velocity field is helicoidal, i.e. a secondary flow occurs. An analytical solution is obtained, without invoking a linear equation of state, but assuming a more general exponential relation between density and temperature. This equation of state reduces to the usual linear relation when very small temperature differences are present within the fluid.

2. The necessary condition for the existence of parallel flows

Let us consider an inclined duct whose cross section has an arbitrary shape. Moreover, let us choose Cartesian coordinates (x, y, z) such that the z-axis is parallel to the duct axis, while the duct cross section lies on the plane (x, y). In particular, the duct cross section corresponds to a region $\mathfrak{D}$ with boundary $\partial\mathfrak{D}$ on the plane (x, y). According to the Boussinesq approximation, for a stationary flow of a Newtonian fluid, the mass balance equation and the momentum balance equation can be expressed as

$$\nabla \cdot \mathbf{U} = 0, \tag{1}$$

$$\varrho_0 \mathbf{U} \cdot \nabla \mathbf{U} = \varrho(T)\mathbf{g} - \nabla p + \mu \nabla^2 \mathbf{U}. \tag{2}$$