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Forced convection with viscous dissipation in the thermal entrance region of a circular duct with prescribed wall heat flux

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Abstract

The thermal entrance forced convection in a circular duct with a prescribed wall heat flux distribution is studied under the assumptions of a fully developed laminar flow and of a negligible axial heat conduction in the fluid, by taking into account the effect of viscous dissipation. The solution of the local energy balance equation is obtained analytically by employing the Laplace transform method. The effect of viscous dissipation is taken into account also in the region upstream of the entrance cross-section, by assuming an adiabatic preparation of the fluid. The latter hypothesis implies that the initial condition in the entrance cross-section is a non-uniform radial temperature distribution. Two special cases are investigated in detail: an axially uniform wall heat flux, a wall heat flux varying linearly in the axial direction.

Keywords: Forced convection; Duct flow; Viscous dissipation; Analytical methods; Laplace transform

1. Introduction

The effect of viscous dissipation may become very important in several flow configurations occurring in the engineering practice. In fact, viscous dissipation affects strongly the heat transfer process whenever the operating fluid has a low thermal conductivity, a high viscosity and flows in ducts with a small cross-section and a small wall heat flux. All these features may occur, for instance, in the microchannel flows considered for the design of MEMS. As is well known, the effect of viscous heating increases with the square of the mass flow rate and, as a consequence, becomes specially important under conditions of forced convection.

A traditional arena for predictions of the viscous dissipation effect in duct flows is the analysis of the laminar thermal entrance regime. Several duct geometries have been investigated, even if most of the published papers refer to the circular duct or to the parallel-plate channel. The thermal entrance problem with viscous dissipation has been investigated by Brinkman [1] with reference to uniform wall temperature or adiabatic wall boundary conditions. Further analyses have been performed by Ou and Cheng [2–4], Lin et al. [5] and Basu and Roy [6]. The latter papers include the study of the boundary conditions of uniform wall heat flux [2,6] and of external convection (third kind boundary condition) [5]. The solutions presented by these authors are extensions of the classical Graetz–Nusselt solution, obtained in the absence of internal heat source terms and widely treated in the literature [7]. The main consequence of the viscous dissipation effect is in the evaluation of the local Nusselt number. Indeed, it has been pointed out that this quantity may become singular at some

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Nomenclature

a, b	arbitrary complex numbers, Eq. (35)
Br	$\mu u_m^2/(r_0 q_{w0})$, Brinkman number
c_p	specific heat at constant pressure
c_0	constant, Eq. (65)
C	function of β, Eq. (28)
${}_1F_1$	confluent hypergeometric function of the first kind
G	function of ζ and β, Eq. (37)
k	thermal conductivity
L_{th}^*	dimensionless thermal entrance length
Nu	Nusselt number
Pe	Peclet number
q_w	wall heat flux
q_{w0}	uniform wall heat flux
r	radial coordinate
r_0	radius of the duct
s	Laplace transformed axial coordinate
s_n	simple poles of $\tilde{\psi}(\eta, s)$, Eq. (29)
T	temperature
T_e	entrance wall temperature
u_m	average velocity
Y	analytic function defined by Eq. (35)
z	axial coordinate

Greek symbols

α	thermal diffusivity
β	complex variable, Eq. (26)
β_n	positive roots of Eq. (30)
ζ	dimensionless axial coordinate, Eq. (6)
ζ_s	value of ζ corresponding to a singularity of Nu
η	dimensionless radial coordinate, Eq. (6)
θ	dimensionless temperature, Eq. (6)
μ	dynamic viscosity
ν	kinematic viscosity
ξ	dummy integration variable
φ	function of η and β, Eq. (27)
ϕ_w	dimensionless wall heat flux, Eq. (6)
χ	function of η, Eq. (62)
ψ	function of η and ζ, Eq. (20)
Ψ	function of η and ζ, Eq. (11)
Ω_1, Ω_2	functions of ζ, Eqs. (51) and (70)

Superscripts/subscripts

$\sim$	Laplace transform
$'$	derivative of a function with respect to its argument
w	wall value
b	bulk mean value, Eq. (38)

axial station. The reason of these singularities is that, at some specific positions, the wall heat flux may be nonzero while the difference between the wall temperature and the bulk temperature is zero. Stated differently, the traditional definition of the local Nusselt number based on the choice of the bulk temperature as the reference temperature may become pathologic when viscous heating is taken into account [5]. More recently, studies of the viscous dissipation effect in laminar duct flows have been performed in order to include the cases of slug velocity profile, of slip-flow in microtubes and of non-Newtonian fluid behavior [8–11].

The aim of the present paper is to perform an analytical study of the thermal entrance region heat transfer in a circular duct with a prescribed axially-varying wall heat flux. The effect of viscous dissipation is taken into account in a self-consistent way by assuming a non-uniform temperature profile as the initial condition at the entrance axial station. The latter profile is obtained as the fully developed profile determined by an upstream adiabatic preparation of the fluid. It will be shown that this assumption induces strong differences with respect to the classical solutions of the entrance problem with viscous dissipation, which are based on a uniform entrance temperature profile. The solution of the local energy balance equation is obtained analytically by means of the Laplace transform method.

2. Mathematical model

Let us consider laminar Poiseuille flow in a circular duct such that, in the region $z < 0$, the wall is thermally insulated while, in the region $z > 0$, an axisymmetric wall heat flux distribution $q_w(z)$ is prescribed. A sketch of the duct and of the prescribed boundary conditions is given in Fig. 1. Forced convection regime is considered, the effect of axial heat conduction in the fluid and in the wall is neglected, while the effect of viscous dissipation is taken into account.

Under the above assumptions, the governing equations are

$$2\left(1-\frac{r^2}{r_0^2}\right)\frac{\partial T}{\partial z} = \frac{\alpha}{u_m r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{16 \nu u_m}{c_p r_0^4} r^2; \tag{1}$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0 \quad \text{(symmetry condition)}; \tag{2}$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=r_0} = 0, \quad z < 0; \tag{3}$$