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Applying neural networks to the solution of forward and inverse heat conduction problems

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Abstract

This paper employs the continuous-time analogue Hopfield neural network to compute the temperature distribution in forward heat conduction problems and solves inverse heat conduction problems by using a back propagation neural (BPN) network to identify the unknown boundary conditions. The weak generalization capacity of BPN networks is improved by employing the Bayesian regularization algorithm. The feasibility of the proposed method is examined in a series of numerical simulations. The results show that the proposed neural network analysis method successfully solves forward heat conduction problems and is capable of predicting the unknown parameters in inverse problems with an acceptable error.

Keywords: Neural network; Inverse heat conduction problems (IHCPs)

1. Introduction

In forward heat conduction problems the heating characteristics, the boundary conditions and the initial conditions of a body are known and are used to establish the internal temperature field. Conversely, in inverse heat conduction problems (IHCPs), experimental temperature measurements are taken at various points in the interior of a body and are used to estimate the unknown boundary conditions existing at the external surface. IHCPs are mathematically ill-posed in the sense that the existence, uniqueness and stability of their solutions cannot be assured [1]. IHCPs are generally solved using some form of numerical technique. Classical approaches include space marching [2,3] the single future time step method [1, pp. 115–119], the function specification method [4,1, pp. 119–134], the regularization method [1, pp. 134–145] and the trial function method [1, pp. 145–148]. Since the 1970s, computer science and technology have advanced rapidly and hence contemporary researchers generally solve IHCPs using numerical methods such as the finite element method [5–7], the finite different method (FDM) [8], the boundary element method [9,10], sequential method [11], Kalman filter method [12,13] and Genetic algorithm [14].

In examining forward heat conduction problems, this study commences by developing the governing equations for various one- and two-dimensional heat conduction cases and then describes the continuous-time analogue Hopfield neural network (CHNN) scheme. The differential equations of the CHNN are derived and correlated with the governing equations of the conduction problems. The framed Hopfield-type neural network is then applied to solve a number of conventional, and rather more complicated, one- and two-dimensional heat conduction problems. The accuracy of the CHNN solutions is verified via comparison with the exact solutions and the FDM results.

The rapid development of artificial neural network technology in recent years has led to an entirely new approach for the solution of IHCPs [15–18,36]. Neural networks are artificial intelligence systems which mimic the biological

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Nomenclature

BPN	back propagation neural
C_i	amplifier input capacitance of ith neuron
CHNN	Continuous-time analogue Hopfield neural network
d_j	desired output of jth neuron
I_i	external input to ith neuron
L	thermal layer thickness
M, N	total number of spatial nodes
q	heat flux
R_i	resistance of ith neuron
t	time (continuous)
t_f	final time
T	temperature
T_0	uniform initial temperature
$\widehat{T}_i$	difference between temperature of ith neuron and its neighbor
u_i	internal state of ith neuron
$\bar{\mathbf{w}}_1, \bar{\mathbf{w}}_2, \bar{\mathbf{w}}_3$	sub-matrices of $\mathbf{W}$
w_{ij}	connection strength between neurons j and i
$\mathbf{W}$	coefficient matrix
$\tilde{x}, \tilde{y}$	dimensionless axial coordinate
$\tilde{x}_u, \tilde{y}_u$	measurement location
y_j	output of jth neuron
α	thermal diffusivity
β	steepness of sigmoid function
δt	sampling interval time
ξ_1, ξ_2	object function parameters
θ_k	bias of neural network
κ	thermal conductivity
ρ	total shunt capacitance
$\varphi(\cdot)$	neuron activation function
Ψ	coefficient matrix
Ψ_j	continuous functions of one variable
$\overline{\Psi}_1, \overline{\Psi}_2$	sub-matrices of Ψ
o	null matrix

Subscripts

i, j	indices

Superscripts

T	transpose of matrix
$\sim$	dimensionless value

processes of a human brain by using non-linear processing units to simulate the functions of biological neurons. The processing units (nodes) are fully interconnected by joints of invariable strength which mimic the synaptic behavior of the human brain. The units are multi-dimensional, self-organizing, fuzzy and self-learning capabilities. Neural networks can be applied to the solution of IHCPs by training the network with multiple samples of temperature distribution data obtained from forward heat conduction problems and adjusting the weights of the individual nodes such that the actual network outputs closely approximate the target values. The fully trained neural network can then predict an unknown output for any arbitrary input by applying the network weights established during the training stage.

The following sections perform a forward analysis using a neural network. Two inverse heat conduction problems are then considered to confirm the validity of the proposed method. The first problem concerns a one-dimensional cylindrical coordinate system, while the second involves a two-dimensional rectangular coordinate system. In both cases, the CHNN scheme is used to perform the forward heat conduction analysis. The results of the forward analysis are used as training data for a three-layered back propagation neural (BPN) network designed to solve IHCPs with different heat profiles. The BPN network is trained using eight different algorithms and the relative performance of each algorithm is examined in terms of its convergence rate and the accuracy of the final solutions. These algorithms include: Conjugate gradient back propagation with resilient back propagation (CRB) [19], gradient descent with momentum and adaptive learning rate back propagation (GMB) [20], conjugate gradient back propagation with Fletcher–Reeves updates (CBF) [21], scaled conjugate gradient back propagation (SCB) [22], quasi-Newton back propagation (QNB) [21, p. 242], one-step secant back propagation (OSB) [23], conjugate gradient back propagation with Powell–Beale restarts (CBP) [24] and Levenberg–Marquardt back propagation (LMB) [20]. To overcome the weak generalization capacity of general back propagation algorithms when applied to non-linear function approximations, and to take account of the uncertain noise inherent in the current IHCPs, the network is also trained using the Bayesian regularization scheme. The performance of the network trained using the best training algorithm is then compared with that trained using the Bayesian regularization approach.

2. Formulation of forward heat conduction problems

This study applies the CHNN model to solve the temperature distribution field of various forward heat conduction problems expressed in either a one-dimensional Cartesian coordinate system, a cylindrical coordinate system or a two-dimensional rectangular coordinate system.

2.1. One-dimensional Cartesian coordinate system

Initially, this study constructs the homogeneous differential equation for the one-dimensional heat conduction case. It is assumed that the one-dimensional bar (in Fig. 5a top