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International Journal of Heat and Mass Transfer 49 (2006) 4886–4892
International Journal of HEAT and MASS TRANSFER
www.elsevier.com/locate/ijhmt

Exclusion of oscillations in heterogeneous and bi-composite media thermal conduction

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Received 25 January 2006; received in revised form 21 May 2006
Available online 24 August 2006

Abstract

Analysis of Fourier heat conduction in heterogeneous and bi-composite media (e.g. porous media, fluid suspensions, etc.) subject to Lack of Local Thermal Equilibrium (LaLotheq) reveals a condition for thermal oscillations and resonance to be possible. This paper shows that this condition cannot be fulfilled because of physical constraints leading to the exclusion of thermal waves and resonance.

Keywords: Heterogeneous media; Bi-composite media; Porous media conduction; Local thermal equilibrium; Local thermal non-equilibrium; Thermal waves; Lotheq; LaLotheq

1. Introduction

Previous analyses [1,2] showed that the physical conditions necessary for thermal waves to be possible in porous media heat conduction subject to Lack of Local thermal equilibrium (LaLotheq) cannot be fulfilled by a Dual-Phase-Lagging (DuPhlag) approximation of the two phase conduction process for a rectangular slab subject to a combination of Dirichlet–Dirichlet [1] or Dirichlet–Newmann [2] set of boundary conditions. The present paper demonstrates that for a combination of Dirichlet–Dirichlet boundary conditions the exclusion of oscillations and consequently resonance is anticipated in the general case and not only in the Dual-Phase-Lagging (DuPhlag) approximation limit. The results apply also not only to porous media but to any heterogeneous system consisting of two phases, such as fluid suspensions [3], or bi-composite materials consisting of a combination of two different solid phases. When both phases are interconnected the derivations presented below apply accurately, while for the case when one phase is continuous and the other is dispersed (such as solid particles suspended in a fluid) the Dual-Phase-Lagging (DuPhlag) formulation applies accurately and not merely as an approximation as demonstrated by Vadasz [3]. In the latter case the DuPhlag results presented by Vadasz [1,2] that are excluding thermal waves are also accurately applicable.

The system of governing equations for Fourier conduction in porous media subject to Lack of Local Thermal Equilibrium (LaLotheq) was showed by Tzou [4] to be approximately equivalent to the Dual-Phase-Lagging (DuPhlag) model of heat conduction. The latter can produce thermal waves in the form of oscillations (see [4]). As a result the Dual-Phase-Lagging (DuPhlag) model can yield thermal resonance when periodically forced by a periodic heat source or a periodic boundary condition with a forcing frequency that is in the neighbourhood of one of the natural frequencies of the system. Tzou [4–6] presents applications of the DuPhlag model to a wide variety of fields from ultrafast (femtosecond) pulse-laser heating of metal films, phonon–electron interaction at nano and micro-scale heat transfer, temperature pulses in superfluid

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doi:10.1016/j.ijheatmasstransfer.2006.05.034

Nomenclature

Bh	bi-harmonic dimensionless group, β_e/L^2
Bf	bi-harmonic-Fourier dimensionless group, Bh/Fo_q
c^2	speed of propagation of the thermal wave, τ_q/α_e (dimensional)
$c_{p,f}$, c_s	fluid and solid phase specific heat, respectively (dimensional)
c_n	dimensionless damping coefficient defined by Eq. (31)
Fo_q	heat flux related Fourier number, $\alpha_e\tau_q/L^2$
Fo_T	temperature gradient related Fourier number, $\alpha_e\tau_T/L^2$
h	integral heat transfer coefficient for the heat conduction at the solid–fluid interface (dimensional)
k_s	effective thermal conductivity of the solid phase, $(1-\varphi)\tilde{k}_s$ (dimensional)
$\tilde{k}_s$	thermal conductivity of the solid phase, (dimensional)
k_f	effective thermal conductivity of the fluid phase, $\varphi\tilde{k}_f$ (dimensional)
$\tilde{k}_f$	thermal conductivity of the fluid phase, (dimensional)
L	the length of the slab (dimensional)
$\boldsymbol{q}$	heat flux vector (dimensional)
t_*	time (dimensional)
T	temperature, (dimensional)
T_C	coldest wall temperature (dimensional)
T_H	hottest wall temperature (dimensional)
x_*	horizontal co-ordinate (dimensional)

Greek symbols

α_e	effective thermal diffusivity, defined by Eq. (5), (dimensional)
β_e	bi-harmonic coefficient, defined in Eq. (5) (dimensional)
γ_s	solid phase effective heat capacity, $(1-\varphi)\rho_s c_s$ (dimensional)
γ_f	fluid phase effective heat capacity, $\varphi\rho_f c_{p,f}$ (dimensional)
θ	dimensionless temperature, $(T-T_C)/(T_H-T_C)$
φ	porosity
ρ_s	solid phase density
ρ_f	fluid phase density
τ_q	time lag associated with the heat flux, defined by Eq. (5), (dimensional)
τ_T	time lag associated with the temperature gradient defined by Eq. (5), (dimensional)
ω_n	dimensionless natural thermal frequency defined by Eq. (31)
ψ	time lags ratio defined by Eq. (10)

Subscripts

*	corresponding to dimensional values of the independent variables, except for cases where there is no ambiguity, as listed in this nomenclature
s	related to the solid phase
f	related to the fluid phase

liquid helium, thermal lagging in amorphous materials, and thermal waves under rapidly propagating cracks.

Analytical solutions as well as analysis of the DuPhlag heat conduction were presented among others in excellent papers by Xu and Wang [7], Wang et al. [8], and Wang and Xu [9] and Antaki [10].

Applications of porous media heat transfer subject to Lack of Local Thermal Equilibrium (LaLotheq) were undertaken among others by Nield [11], Minkowycz et al. [12], Banu and Rees [13], Baytas and Pop [14], Kim and Jang [15], Rees [16], Alazmi and Vafai [17], and Nield et al. [18]. While the significance of practically obtaining the same temperature solution for each phase in a porous medium subject to a Lack of Local Thermal Equilibrium (LaLotheq) is discussed by Vadasz [19] identifying conditions for which the traditional formulation of the LaLotheq model might not be adequate, the conditions used in the present paper are not identical to those identified by Vadasz [19].

The present paper deals with Fourier heat conduction in a heterogeneous (e.g. porous) or bi-composite medium subject to LaLotheq. The latter produces a bi-harmonic linear partial differential equation that possesses wave properties. Nevertheless, physical constraints exclude the possibility of thermal wave solutions in such systems. The present paper demonstrates this exclusion for a heterogeneous (e.g. porous) or bi-composite slab subject to a combination of Dirichlet–Dirichlet boundary conditions.

2. Problem formulation and properties of the LaLotheq system

The following analysis uses the terminology applicable to heat conduction in porous media, although it applies equally well to any other heterogeneous or bi-composite system. Therefore the terminology of "solid phase–fluid phase" should be converted to "solid-phase 1–solid phase 2" in the case of bi-composite systems and similar conversions apply to other two-phase systems. The heat conduction equations for the two phases that compose an isotropic and homogeneous porous medium subject to LaLotheq are obtained as phase averages over a Representative Elementary Volume (REV) following *Fourier's Law* in the form: