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Thermodynamic optimization of tree-shaped flow geometries

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Abstract

In this paper we optimize the performance of several classes of simple flow systems consisting of T- and Y-shaped assemblies of ducts, channels and streams. In each case, the objective is to identify the geometric configuration that maximizes performance subject to several global constraints. Maximum thermodynamic performance is achieved by minimization of the entropy generated in the assemblies. The boundary conditions are fixed heat flow per unit length and uniform and constant heat flux. The flow is assumed laminar and fully developed. Every geometrical detail of the optimized structure is deduced from the constructal law. Performance evaluation criterion is proposed for evaluation and comparison of the effectiveness of different tree-shaped design heat exchangers. This criterion takes into account and compare the entropy generated in the system with heat transfer performance achieved.

Keywords: Constructal theory; Tree-shaped flow; Thermodynamic optimization; Entropy generation minimization

1. Introduction

Among the more recent methods that have become established in thermal engineering, thermodynamic optimization has the objective of improving the global performance of the system subject to specified global constraints. Thermodynamic optimization is useful as a first step, for orientation in the search of tradeoffs that govern the geometrical configuration of the system. Tree networks represent a new trend in the optimization and miniaturization of heat transfer devices [1–6], mass exchangers [7,8], chemical reactors [9], and fuel cells [10–12]. Tree-shaped architectures promise a more judicious use of the available space: higher densities of heat and mass transfer and chemical reactions, and a more uniform volumetric distribution of transport processes. The fundamental study of the optimization of tree-shaped architectures also sheds light on the common design principles of engineered and natural flow systems.

In design, and in society in general, space is at a premium. This is why the interest in performance at smaller and smaller scales is natural, and will continue. The miniaturization revolution means not only that the smallest identifiable volume element (the *elemental* system [1]) is becoming smaller, but also that larger and larger numbers of such elements must inhabit the microscopic device that they serve. The smaller the elements, and the larger their number, the greater the complexity of the structure. In design, miniaturization also means increasing complexity. Packing the system with smaller, more powerful and more numerous elemental systems is a necessary first step. The challenge is not only to find geometric arrangements to connect the currents that access the elemental systems, but to *optimize* each connection such that, ultimately, each design choice is reflected in an increase in performance at the global level. To assemble more and more elements into complex structures, and to optimize (with global objective and space constraints) each connection means to *construct*.

Improvement in the global thermodynamic performance of a system means the decrease in the irreversibility

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Nomenclature

A	area (m^2)
c_p	specific heat ($J\ kg^{-1}\ K^{-1}$)
D	channel diameter (m)
f	Fanning friction factor
h	heat transfer coefficient ($W\ m^{-2}\ K^{-1}$)
k	thermal conductivity ($W\ m^{-1}\ K^{-1}$)
L	length (m)
M	dimensionless mass flow rate, $M = \dot{m}c_p/\left(\pi k Nu A^{1/2}\right)$
$\dot{m}$	mass flow rate ($kg\ s^{-1}$)
N_s	entropy generation ratio, $N_s = T\dot{S}_{gen}/q$
Nu	Nusselt number, $Nu = h_i D_i/k$
n	number of pairing levels
n_0	number of central ducts
P	pressure (Pa)
q	heat flow (W)
q'	heat flow per unit length ($W\ m^{-1}$)
q''	heat flux ($W\ m^{-2}$)
$\tilde{q}'$	dimensionless heat flow per unit length, $\tilde{q}' = q'/(\pi k Nu T)$
$\tilde{q}''$	dimensionless heat flux, $\tilde{q}'' = q'' A^{1/2}/(\pi k Nu T)$
r	radius (m)
$\dot{S}_{gen}$	entropy generation rate ($W\ K^{-1}$)
$\widetilde{S}_{gen}$	entropy generation number
T	temperature (K)
ΔT	temperature difference (K)
V	volume (m^3)
$\dot{W}$	pumping power (W), $\dot{W} = \dot{m}\Delta P/\rho$
$\widetilde{W}$	dimensionless pumping power, $\widetilde{W} = \dot{W}V^2/\left[(kNu/c_p)^2(\nu/\rho)A^{5/2}\right]$

Greek symbols

ν	kinematic viscosity ($m^2\ s^{-1}$)
ρ	density ($kg\ m^{-3}$)
τ	$\Delta T/T$

Subscripts

i	inlet or channel rank
m	mean
n	number of construction levels
o	outlet

(or entropy generation, exergy destruction) that characterizes all the components and processes of the system. An engineering flow system owes its irreversibility to several mechanisms, most notably the flow of heat, fluid and electric current due to driving potentials, and against finite resistances. The entropy generated by each current is proportional to the product of the current times the driving potential, i.e., proportional to the resistance overcome by the current. In simple terms, the entire effort to optimize thermodynamically the greater system rests on the ability to minimize all internal flow resistances, together. Because of constraints, the resistances compete against each other.

The route to improvements in global performance is by *balancing* the reductions in the competing resistances. Thermodynamically, this amounts to spreading the entropy generation rate through the system in an optimal way, so that the total irreversibility is reduced. Optimal spreading of imperfection is achieved by properly sizing, shaping and positioning the components. In the end, the geometry structure of the system—its architecture—emerges as a result of global thermodynamic optimization.

Tree-shaped flows have been studied extensively recently [11–19]. Bejan [20], and da Silva et al. [21] proposed to use dendritic flow architecture in the conceptual design of two-stream heat exchangers. This is a new direction for the development of the heat exchanger architecture. The ultimate goal is to determine flow architectures that reach *simultaneously* two objectives: (i) minimal global fluid resistance (or pumping power), and (ii) minimal thermal resistance. When the architecture is optimized for (i), the result is a dendritic structure in which every geometric feature is uniquely determined. The corresponding thermal resistance decreases as the total mass flow rate and pumping power increase. When the objective is (ii), the optimal architecture has radial ducts, not dendrites. The corresponding fluid-flow resistance increases as the flow rate increases and the global thermal resistance decreases.

In this paper we propose a new way of approaching the geometric optimization of tree-shaped paths for fluid flow. The objective is to determine flow architectures that reach simultaneously two objectives: (i) minimal global entropy generated, and (ii) maximum heat flow density. We consider simple building block consisting of a few streams that serve as tributaries or branches in a constrained space. A larger stream with two branches (or two tributaries) forms a construct shaped as T or Y. We also show that putting together the optimized constructs it is possible to reconstruct features of the much more complicated tree structures optimized so far. Next, we show a performance evaluation criterion for evaluation of the performance of new tree-shaped flow geometries through comparison of the entropy generated in the system with the heat transfer performance achieved.

2. Boundary condition: specified heat flow per unit length

2.1. Problem formulation

In order to calculate the entropy generation, we consider an axially uniform duct of circular cross-section with a