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Solution of radiative heat transfer in a semitransparent slab with an arbitrary refractive index distribution and diffuse gray boundaries

He-Ping Tan *, Yong Huang *, Xin-Lin Xia

School of Energy Science and Engineering, Harbin Institute of Technology, 92 West Dazhi Str., Harbin, Heilongjiang 150001, China

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Abstract

On the basis of medium discretization and local linear approximation of refractive index distribution, the curved ray tracing technique is used in combination with the pseudo source adding method to numerically solve the radiative heat transfer in a semitransparent slab with an arbitrary refractive index distribution and two diffuse gray walls. The radiative equilibrium temperature field of a linear refractive index distribution is evaluated by this method and the results show excellent agreement with that of the previous research. For two types of sinusoidal refractive index distributions, the radiative equilibrium temperature field as well as the temperature and heat flux fields of coupled radiation–conduction are investigated in detail. The results show considerable significance of the gradient refractive index effect, and some important conclusions are to be obtained.

Keywords: Radiation transfer; Gradient refractive index; Semitransparent medium; Curved ray tracing technique; Pseudo source adding method

1. Introduction

The radiative heat transfer in a gradient refractive index medium can be found in many technology processes, such as the heating of glass and thermal protecting coatings, the manufacturing of waveguide materials, the ray transporting through atmosphere, as well as the optical measurement of flame and other semitransparent medium. But the investigation on such heat transfer in a gradient refractive index medium has not been found until recent years. In 2000, Ben Abdallah and Le Dez put forward a curved ray tracing technique to solve the radiation transfer in a gradient refractive index medium, and first analyzed the thermal emission of a semitransparent slab with variable spatial refractive index [1]. By this method, they also investigated the radiation transfer and coupled radiation–conduction inside a semitransparent slab with variable spatial refractive index [2,3], as well as the thermal emission of a two-dimensional rectangular cavity [4]. In 2002, Xia and Huang et al. analyzed the thermal emission and volumetric absorption of a graded index semitransparent medium layer by a ray splitting and tracing technique [5], and resorting to a pseudo source adding method to deal with the radiative intensity on boundary surfaces, the curved ray tracing technique is used to solve the radiation transfer and the couple radiation–conduction in a gradient refractive index slab with gray boundaries [6,7]. Also in 2002, Lemonnier and Le Dez presented a discrete ordinate solution of radiative transfer across a slab with variable refractive index [8]. All these researches have show the considerable effect of gradient refractive index on radiation transfer in medium.

In this paper, the radiative heat transfer in an absorbing-emitting semitransparent slab with an arbitrary

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* Corresponding authors. Tel.: +86-451-641-2148.
E-mail address: huangy_zl@263.net (Y. Huang).

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Nomenclature

a_k	coefficient denoting the influence of temperature of sublayer k on $I^-(0,\xi)$
A_k	coefficient denoting the influence of temperature of sublayer k on I_{01}
b, b'	coefficients denoting the influence of sublayer temperature on $I(z_j,\xi)$and $I(z_j,-\xi)$, respectively
B_{kj}, B'_{kj}	coefficients defined in Eqs. (27) and (28), respectively
c_k	coefficient denoting the influence of temperature of sublayer k on $I^+(d,\xi)$
C_k	coefficient denoting the influence of temperature of sublayer k on I_{d1}
D_{kj}	coefficients defined in Eq. (36)
d	slab thickness, m
I	radiative intensity, W/m^2 sr
$I^*(j)$	reduced radiative intensities ($n=1$) at the jth splitting point, W/m^2 sr
K_{ij}	influencing factor denote the coupling influence of the two boundaries
L	number of discrete angles of the hemispherical space
$n(z)$	refractive index distribution
$\tilde{n}(z)$	linear approximation of refractive index distribution in a sublayer
M	number of sublayers
N	radiative–conductive parameter, $N=\lambda\kappa/(4\sigma n_{\mathrm{R}}^2 T_{\mathrm{R}}^3)$, $n_{\mathrm{R}}=(n_{\max}+n_{\min})/2$
q	heat flux density, W/m^2
q^*	dimensionless heat flux, $q^*=q/q_{\mathrm{R}}$, $q_{\mathrm{R}}=\sigma T_{\mathrm{R}}^4$
$s(j)$	curve length of the propagating route between two tracing points
T	temperature, K
T_{R}	reference temperature, $T_{\mathrm{R}}=(T_0+T_d)/2$, K
z	space coordinate, m
Δz	sublayer thickness, m

Greek symbols

ε	emissivity of boundary wall
ζ	angle between the ray propagating direction and the interface normal
κ	absorption coefficient, m^{-1}
λ	thermal conductivity, W/m K
ξ	polar angle of propagating direction
σ	Stefan–Boltzmann constant, 5.729×10^{-8} W/m^2 K^4
τ	optical thickness, $\tau=\kappa d$
Ω	spatial direction

Superscripts and subscripts

$+,-$	direction from boundary 2 to boundary 1 ($-$) or from boundary 1 to boundary 2 ($+$)
c	conduction
d	position of $z=d$
j, k	sublayer order
0	position of $z=0$
p	pseudo source
r	radiation
t	total radiation and conduction

refractive index distribution and diffuse gray boundaries is to be solved numerically. For two types of sinusoidal refractive index distributions, the radiation transfer and coupled radiation–conduction are analyzed in detail.

2. Geometrical and physical model

Consider an infinite parallel plane slab of absorbing–emitting but non-scattering gray medium of thickness d, as shown in Fig. 1. The emissivities of the two diffuse gray boundary walls are ε_0 and ε_d , and the temperatures are T_0 and T_d, respectively. The medium is characterized by a constant absorption coefficient κ, a constant thermal conductivity λ and a refractive index distribution $n(z)$. A steady state temperature field $T(z)$ will be caused by the coupled radiation–conduction in medium. Radiation will dominate the heat transfer process if the thermal conductivity λ is small enough, and the temperature field would be a radiative equilibrium one thereby.

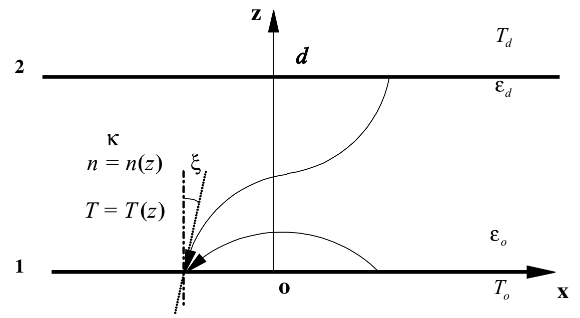

Fig. 1. Schematic diagram of geometrical and physical model.

3. Discretization and solution

3.1. Discretization

As shown in Fig. 2, the medium is divided into $M-2$ isothermal sublayers of equal thickness Δz. The two