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Dimensionless and analytical studies of the thermal stability of a high temperature superconducting tube

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Abstract

This paper deals with the influence of thermal conductivity and specific heat on the stability of a superconducting tube. The study is made to foresee the effect of a pulse of energy, which causes the quench of the superconductor. Depending on the applications, the transition of the superconductor is provoked, like for the superconducting current limiter, or accidental. In any case, it is necessary to protect the superconductor from an excessive rise of temperature to avoid its destruction. Using dimensionless analysis, the study is applied to a cylindrical tube.

Keywords: High T_c superconductor (A); Stability (C); Heat Transfer (C)

1. Introduction

Superconductors are characterised by two fundamental properties, they have no resistance and are perfectly diamagnetic. This diamagnetism allows to expel or to trap magnetic field. These properties exist only under some conditions on temperature T, magnetic field H and current density J. In this 3-D space, three critical values T_c, H_c and J_c, define a critical surface for a given material. This material is in superconducting state if, and only if, its functioning point is located under the so defined surface.

When a local energy is supplied in a superconducting material, its temperature increases. If the temperature rise is sufficient, a part of the superconductor becomes resistive. Therefore, two cases are to be considered: either the normal zone vanishes and the system is stable or it expands to the whole system and the system is unstable. This transition could be useful in some uses to limit the current or to give a pulse of energy, but it must be avoided in other applications. Whatever the application, the study of the transition is necessary to prevent an excessive temperature rise of the superconductor for fear of destruction.

High critical temperature materials, which appear in 1986, will be used, without any doubt, in many areas, and especially in electrical engineering. Today, those new materials are mainly ceramics and can be used, in some applications, under a bulk shape.

To contribute to the expansion of this material, besides theoretical studies, engineering tools have to be developed to help the designer of such apparatus. This paper is a contribution to the study of the important problem of the stability of the bulk superconductor.

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The study presented here concerns the influence of the geometrical and physical parameters on the stability of a tube shaped superconductor. This form is used in many applications like current lead or cylindrical current limiter.

Taking into account the fixed goal, we have to set methods in which the parameters appear explicitly. To do that, we set some hypothesis to simplify the models.

The methods presented as well as the numerical results could be extended to other superconducting material or to other shapes.

The paper is divided into three main parts; the first one is devoted to the model, the hypothesis and the analytical solutions. Numerical results are developed in the second part. The variations of the physical parameters with the temperature are presented in the last part.

2. Modelling

The studied system is a tube made of a bulk superconducting material and usually used as a current lead. All the geometrical parameters are reported in Fig. 1. Of course, because of the shape, cylindrical co-ordinates are used to set the mathematical model.

We study the behaviour of this tube when an energy pulse is applied for a short time in a small zone in its thickness. The considered impulsion can be provoked by an extra current, by a shock or by a structure default.

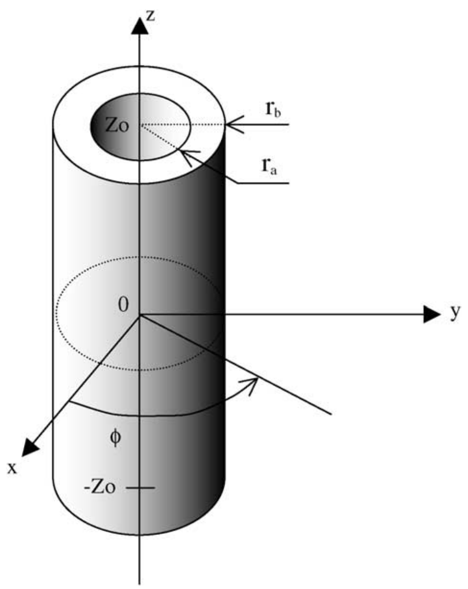

Fig. 1. Representation of the tube.

2.1. Limits of the study and assumptions

The study will be limited to a short time following the pulse application, this time is sufficient to know if the system goes to a stable state or not. The shortness of the time implies that the heat cannot diffuse to the borders of the tube, so the temperature can be considered as adiabatic on the walls. So, the conditions of the study are the followings:

- $T = T_{\text{bath}}$ for the whole tube at $t = 0$.
- $$\frac{\partial T}{\partial n} = 0 \text{ on the boundaries } \quad \forall t \tag{1}$$
- The current in the tube is constant because it is usually imposed by an external source.
- Only the own field of the tube is considered to the exclusion of any other external field.

2.2. Heat transfer

Temperature distribution $T(r, \theta, z, t)$ is governed by heat transfer equation:

$$C_p \frac{\partial T}{\partial t} = \operatorname{div}\left(\overline{\overline{\Lambda}}\vec{\nabla}(T)\right) + P \tag{2}$$

where P is an internal source, which is the power dissipated by Joule effect (W m^{-3}), $C_p(T)$ is the specific heat (J m^{-3} K^{-1}), $\overline{\overline{\Lambda}}$(T) is the tensor of the thermal conductivity (W m^{-1} K^{-1}).

To simplify this tensor, radial and axial thermal conductivities are expressed as functions of the azimuthal component of thermal conductivity. Introducing coefficients α_r and α_z, this tensor can be expressed as

$$\overline{\overline{\Lambda}} = \begin{pmatrix} \lambda_z & 0 & 0 \\ 0 & \lambda_r & 0 \\ 0 & 0 & \lambda_\theta \end{pmatrix} = \lambda_\theta \begin{pmatrix} 1/\alpha_z & 0 & 0 \\ 0 & 1/\alpha_r & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3}$$

In cylindrical co-ordinates, Eq. (2) becomes:

$$C_p \frac{\partial T}{\partial t} = \lambda_z \frac{\partial^2 T}{\partial z^2} + \lambda_r \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) + \frac{\lambda_\theta}{r^2}\frac{\partial^2 T}{\partial \theta^2} + P \tag{4}$$

and could be rewritten as

$$C_p \frac{\partial T}{\partial t} = \lambda_\theta \left(\frac{1}{\alpha_z}\frac{\partial^2 T}{\partial z^2} + \frac{1}{\alpha_r}\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}\right) + P \tag{5}$$

Now, we introduce a coefficient α_{cp}. Thanks to this coefficient, we are able to vary the specific heat around a prescribed value. So, the last equation is rewritten as following:

$$\alpha_{cp} C_p \frac{\partial T}{\partial t} = \lambda_\theta \left(\frac{1}{\alpha_z}\frac{\partial^2 T}{\partial z^2} + \frac{1}{\alpha_r}\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}\right) + P \tag{6}$$