Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
International Journal of Heat and Mass Transfer 48 (2005) 2822–2828
International Journal of HEAT and MASS TRANSFER
www.elsevier.com/locate/ijhmt

Lack of oscillations in Dual-Phase-Lagging heat conduction for a porous slab subject to imposed heat flux and temperature

Peter Vadasz *

Department of Mechanical Engineering, Northern Arizona University, P.O. Box 15600, Flagstaff, AZ 86011-5600, USA

Received 20 December 2004; received in revised form 3 February 2005
Available online 20 April 2005

Abstract

This study shows that the physical conditions necessary for thermal waves to materialize in Dual-Phase-Lagging porous media conduction are not attainable in a porous slab subject to a combination of constant heat flux and temperature (Neumann and Dirichlet) boundary conditions. It is demonstrated that the approximate equivalence between Dual-Phase-Lagging (DuPhlag) heat conduction model and the Fourier heat conduction in porous media subject to Lack of Local Thermal Equilibrium (La Lotheq) that suggested the possibility of thermal oscillations and resonance reveals a condition that cannot be fulfilled because of physical constraints.

Keywords: Dual-Phase-Lagging; Hyperbolic heat conduction; Local thermal equilibrium; Porous media; Thermal waves

1. Introduction

This is a companion paper to Vadasz [1] as a complementary study aiming at demonstrating that oscillations are not possible in Dual-Phase-Lagging heat conduction in a porous slab subject to a combination of Dirichlet and Neumann boundary conditions. Repetition will therefore be kept to a minimum and presented only for the purpose of consistency and flow of presentation.

The system of governing equations for Fourier conduction in porous media subject to Lack of Local Thermal Equilibrium (La Lotheq) was showed by Tzou [2] to be approximately equivalent to the Dual-Phase-Lagging (DuPhlag) model of heat conduction. The latter can produce thermal waves in the form of oscillations. As a result the Dual-Phase-Lagging (DuPhlag) model can yield thermal resonance when periodically forced by a periodic heat source or a periodic boundary condition with a forcing frequency that is equal to one of the natural frequencies of the system. Tzou [2–4] presents applications of the DuPhlag model to a wide variety of fields from ultrafast (femtosecond) pulse-laser heating of metal films, phonon–electron interaction at nano and micro-scale heat transfer, temperature pulses in superfluid liquid helium, thermal lagging in amorphous materials, and thermal waves under rapidly propagating cracks.

Analytical solutions as well as analysis of the DuPhlag heat conduction were presented among others in excellent papers by Xu and Wang [5], Wang et al. [6], and Wang and Xu [7] and Antaki [8].

Applications of porous media heat transfer subject to Lack of Local Thermal Equilibrium (La Lotheq) were undertaken among others by Nield [9], Minkowycz et al. [10], Banu and Rees [11], Baytas and Pop [12],

* Tel.: +1 928 523 5843; fax: +1 928 523 8951.
E-mail address: peter.vadasz@nau.edu

0017-9310/$ - see front matter
doi:10.1016/j.ijheatmasstransfer.2005.02.005

Nomenclature

$c_{p,\mathrm{f}}$, c_{s}	fluid and solid phase specific heat, respectively (dimensional)
c_n	dimensionless damping coefficient defined by Eq. (26)
$\hat{\boldsymbol{e}}_x$	unit vector in the x direction
$\hat{\boldsymbol{e}}_y$	unit vector in the y direction
$\hat{\boldsymbol{e}}_z$	unit vector in the z direction
Fo_q	heat flux related Fourier number, equals $\alpha_e\tau_q/L^2$
Fo_T	temperature gradient related Fourier number, equals $\alpha_e\tau_T/L^2$
N_β	Dual-Phase-Lagging bi-harmonic term dimensionless group, equals $\beta_e/\alpha_e L^2$
h	integral heat transfer coefficient for the heat conduction at the solid–fluid interface (dimensional)
k_{s}	effective thermal conductivity of the solid phase, equals $(1-\varphi)\tilde{k}_{\mathrm{s}}$ (dimensional)
$\tilde{k}_{\mathrm{s}}$	thermal conductivity of the solid phase (dimensional)
k_{f}	effective thermal conductivity of the fluid phase, equals $\varphi\tilde{k}_{\mathrm{f}}$ (dimensional)
$\tilde{k}_{\mathrm{f}}$	thermal conductivity of the fluid phase (dimensional)
L	the length of the porous slab (dimensional)
$\boldsymbol{q}$	heat flux vector (dimensional)
t_*	time (dimensional)
T	temperature (dimensional)
T_{C}	coldest wall temperature (dimensional)
x_*	horizontal co-ordinate (dimensional)
$\boldsymbol{x}$	position vector, equals $x\hat{\boldsymbol{e}}_x + y\hat{\boldsymbol{e}}_y + z\hat{\boldsymbol{e}}_z$

Greek symbols

α_e	effective thermal diffusivity, defined by Eq. (5) (dimensional)
β_e	effective property coefficient to the Dual-Phase-Lagging bi-harmonic term, defined in Eq. (5) (dimensional)
γ_{s}	solid phase effective heat capacity, equals $(1-\varphi)\rho_{\mathrm{s}}c_{\mathrm{s}}$ (dimensional)
γ_{f}	fluid phase effective heat capacity, equals $\varphi\rho_{\mathrm{f}}c_{p,\mathrm{f}}$ (dimensional)
θ	dimensionless temperature, equals $(T-T_{\mathrm{C}})/(T_{\mathrm{H}}-T_{\mathrm{C}})$
φ	porosity
ρ_{s}	solid phase density
ρ_{f}	fluid phase density
τ_q	time lag associated with the heat flux, defined by Eq. (5) (dimensional)
τ_T	time lag associated with the temperature gradient defined by Eq. (5) (dimensional)
ω_n	dimensionless natural thermal frequency defined by Eq. (26)

Subscripts

*	corresponding to dimensional values of the independent variables, except for cases where there is no ambiguity, as listed in this nomenclature
s	related to the solid phase
f	related to the fluid phase

Kim and Jang [13], Rees [14], Alazmi and Vafai [15], and Nield, Kuzentsov and Xiong [16]. While the significance of practically obtaining the same temperature solution for each phase in a porous medium subject to a Lack of Local Thermal Equilibrium (La Lotheq) is discussed by Vadasz [17] identifying conditions for which the traditional formulation of the La Lotheq model is not adequate, the conditions used in the present paper are not identical to those identified by Vadasz [17]. Other examples of conditions that are not affected by the conclusions of the present paper are problems of convection subject to La Lotheq such as those presented by Spiga and Morini [18], Kuznetsov [19], Amiri and Vafai [20] and Kuznetsov [21].

The present paper deals with Fourier heat conduction in a porous medium subject to La Lotheq. It aims at demonstrating that the condition required for oscillatory solutions, $\tau_T/\tau_q < 1$, is not physically attainable in a porous slab conduction subject to a combination of an imposed constant heat flux (Neumann) and constant temperature (Dirichlet) boundary conditions. While the results of the present paper may provide a useful guidance among others to pulsed laser processing of nanofilms (e.g. [22]), the problem presented here is essentially distinct and deals with the application to porous media. There are major distinctions as well as similarities between the two. The similarities are linked to the two-phase coupled equations used to represent the "absorption of photon energy by electrons and the heating of the lattice through electron–phonon coupling" [23]. The distinctions are mainly in the small scale of the ultra fast heating of metals or thin films leading to a legitimate use of a non-Fourier constitutive model to represent the relationship between the heat flux and the temperature gradient, such as the application of the Dual-Phase-Lagging for each phase (Hays-Stang and Haji-Sheikh [22]). In porous media due to the typical macroscopic scale of both phases the latter is not applicable. In the present paper Fourier Law was employed for the heat flux mechanism at each phase.