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Turbulent forced convection in a helicoidal pipe with substantial pitch

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Abstract—Fully developed turbulent convective heat transfer in a circular cross-section helicoidal pipe with finite pitch is numerically studied. The k-ε model is used to model the turbulent behavior. The time averaged momentum and energy equations are derived in the helicoidal coordinate system. The results indicate that the temperature distribution in the cross-section will be asymmetric as the pitch of the coil increases. Unlike that in laminar flow, an increase in the Prandtl number will reduce the torsion effect on the heat transfer in a helicoidal pipe. The results also indicate that the pitch effect will be enhanced as the flow rate increases. Copyright © 1996 Elsevier Science Ltd.

1. INTRODUCTION

Turbulent heat transfer in coiled pipes is commonly encountered in the design of compact heat exchangers, evaporators, combustors, and condensors used in the food, pharmaceutical, power and chemical industries. The convective turbulent heat transfer in coiled pipes has been experimentally studied by Jeschke [1], Woschni [2], and Rogers and Mayhew [3] for water and air. Petukhov and Popov [4], Schmidt [5] and Gnielinski [6] developed the correlation equations based on the test, and the derivation of the data from the correlation equation was less than $\pm 15\%$. Mori and Nakayama [7] applied boundary layer idealization to predict the temperature profile and heat transfer coefficient for the turbulent flow. Their calculated result was confirmed by their own experimental result.

To distinguish the differences in coiled pipes, the coiled pipe with a negligible pitch is usually termed as a *toroidal* pipe, and the coiled pipe with considerable pitch is designated as a *helicoidal* pipe. Although numerous studies have been conducted for toroidal pipe flow, only a limited number of papers has been published for the flow and heat transfer in a helicoidal pipe (the coiled pipe with considerable pitch). It is well known that the pitch of the coiled pipe will create an additional force—torsion—on the flow. The major obstacle for progress in this area of study is due mainly to the fact that the axial velocity in the helicoidal pipe is not orthogonal to the radial and tangential velocities in a helicoidal coordinate system. Wang [8] first introduced the nonorthogonal helicoidal coordinate to study the secondary flow in a helicoidal pipe. Murata *et al.* [9] simplified the Navier–Stokes equations by assuming a small curvature in a nonorthogonal coordinate system. Germano [10, 11] introduced a transformation to render the nonorthogonal coordinate system to an orthogonal one, and found that the effect of torsion on the secondary flow is second-order. Kao [12] used Germano's coordinate system to study the helicoidal pipe flow in a substantial range of Dean numbers using both perturbation and numerical methods. Recently, Xie [13], Tuttle [14], Chen and Jan [15], and Liu [16] tried to resolve the controversy between these researchers by linking Wang's coordinate system with Germano's coordinate system. Kao [12] and Yang *et al.* [17] studied the convective heat transfer in the helicoidal pipe with a finite pitch. However, all of these studies of the torsion effect on the flow and heat transfer in a helicoidal pipe were limited in the laminar flow region.

The above survey indicates that the theoretical and numerical studies on the forced convective heat transfer in the helicoidal pipe are limited in either the laminar region or the turbulent region with zero pitch. The objective of this study is to explore the effects of torsion on the fully developed turbulent forced convection in a helicoidal pipe with a finite pitch. In this study, the momentum and energy equations were derived in the helicoidal coordinate system as employed by Germano for laminar flow. The k-ε turbulent model, which has been successfully used in the toroidal pipe, has been applied to predict the turbulent viscosity. In the following sections, the governing equations in the helicoidal coordinates and the numerical procedures will be discussed, followed by a discussion of the results and conclusions.

2. THE GOVERNING EQUATIONS

The helicoidal coordinate system used by Germano [10] for laminar flow is applied in this study. In Fig.

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NOMENCLATURE

a	pipe diameter [m]
b	pitch [m]
C_1, C_2, C_μ	constant
De	Dean number
G	turbulent kinetic energy generation [$m^2 s^{-3}$]
k	kinetic energy [$m^2 s^{-2}$]
Pr	Prandtl number
Pr_t	turbulent Prandtl number
p	pressure [$N m^{-2}$]
R	radius of the coil [m]
Re	Reynolds number
r	dimensionless radial direction coordinate [m]
s	dimensional axial coordinate [m]
T	temperature [°C]
u, v, w	velocity components [$m s^{-1}$]
$\mathbf{V}$	velocity [$m s^{-1}$]
w_b	dimensionless average axial velocity [$m s^{-1}$]
y_p	distance from the wall [m].

Greek symbols

α	angle
Γ	turbulent diffusivity
δ	dimensionless curvature, κa
ε	turbulent kinetic energy dissipation [$m^2 s^{-3}$]
θ	angle
κ	constant
λ	dimensionless torsion [$\tau\kappa^{-1}$]
μ	dynamic viscosity [$kg\ m^{-1} s^{-1}$]
ν	kinematic viscosity [$m^2 s^{-1}$]
ρ	density [$kg\ m^{-3}$]
σ_k, σ_ε	constants
τ	torsion; shear stress [$m^2 s^{-2}$]
Φ	general function
ψ	angle
ω	function.

Subscripts

b	bulk
k	turbulent kinetic energy
l	laminar
p	near boundary node
r	radial direction
s	axial direction
T	temperature
t	turbulent
ε	turbulent kinetic energy dissipation
ψ	tangential direction.

1, s indicates the dimensional axial coordinate, and $2a$ is the diameter of the circular cylinder in which s is coiled. R is the radius of the circular pipe, while r and $\psi = \theta + \phi$ are the coordinates in the radial and tangential directions, and b is the pitch of the helicoidal pipe. u, v and w are velocities in the tangential, radial and axial directions, respectively. The governing equations for fully developed flow can be written as

Continuity equation

$$\frac{1}{r}\frac{\partial u}{\partial \psi} + \frac{\partial v}{\partial r} + \frac{v}{r} + \delta\omega\left[u\cos\psi + v\sin\psi - \lambda\frac{\partial w}{\partial \psi}\right] = 0. \quad (1)$$

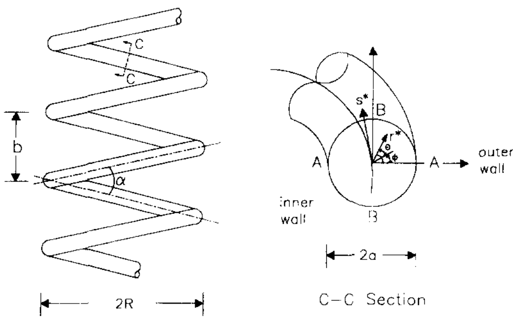

Fig. 1. The schematic of a helicoidal coordinate system.

Momentum equation

$$\rho\left(\frac{\partial(uu)}{r\partial\psi} + \frac{1}{r}\frac{\partial(ruv)}{\partial r}\right)$$
$$= -\frac{1}{r}\frac{\partial p}{\partial \psi} + \rho\delta w\omega\left(w\cos\psi + \lambda\frac{\partial u}{\partial \psi}\right)$$
$$-\rho\frac{uv}{r} - \rho u\delta\omega\left[u\cos\psi + v\sin\psi - \lambda\frac{\partial w}{\partial \psi}\right]$$
$$+\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{r\psi}) + \frac{\partial}{r\partial\theta}(\tau_{\psi\psi}) + \frac{\tau_{r\psi}}{r}$$
$$+\delta\omega(\tau_{\psi\psi}\cos\psi + \tau_{r\psi}\sin\psi + \tau_{ss}\cos\psi)$$
$$-\delta\omega\lambda\frac{\partial}{\partial\psi}(\tau_{\psi s}) \quad (2)$$

$$\rho\left(\frac{1}{r}\frac{\partial(uv)}{\partial\psi} + \frac{1}{r}\frac{\partial(rvv)}{\partial r}\right)$$
$$= -\frac{\partial p}{\partial r} + \frac{1}{r}\frac{\partial(r\tau_{rr})}{\partial r} + \frac{1}{r}\frac{\partial(\tau_{r\psi})}{\partial\psi} + \rho\delta\omega w\Bigg)$$
$$\times\left(w\sin\psi + \lambda\frac{\partial v}{\partial\psi}\right) + \rho\frac{u^2}{r} - \rho v\delta\omega$$