Pergamon

Int. J. Heat Mass Transfer. Vol. 38, No. 11, pp. 2011–2018, 1995

0017-9310/95 $9.50+0.00

0017-9310(94)00313-0

Coriolis effect on free convection in a long rotating porous box subject to uniform heat generation

PETER VADASZ

Department of Mechanical Engineering, University of Durban-Westville, Private Bag X54001, Durban 4000, South Africa

(*Received* 9 *May* 1994 *and in final form* 23 *September* 1994)

Abstract—The Coriolis effect on free convection in a long rotating porous box subject to uniform heat generation is investigated analytically. A three dimensional analytical solution is presented for large values of the porous media Ekman number. The convection results from internal heat generation which produces temperature gradients orthogonal to the centrifugal body force. Two types of thermal boundary conditions are considered for the top and bottom walls of the box. The first type is associated with perfectly conducting boundaries, i.e. the same temperature is imposed on both the top and bottom walls while the second type corresponds to a perfectly conducting top wall and adiabatic bottom wall. The solution to the nonlinear set of partial differential equations is obtained through an asymptotic expansion of the dependent variables in terms of two small parameters representing the reciprocal Ekman number in porous media and the aspect ratio of the domain. Secondary circulation in the form of one or two vortices is obtained in a plane orthogonal to the leading free convection plane.

1. INTRODUCTION

Transport phenomena in rotating porous media have a wide spectrum of applications in engineering and geophysics [1, 2]. The effect of rotation on free convection is of particular interest from both the practical and theoretical points of view.

Research results [3–7] are available for natural convection in rotating porous media resulting from gravity in the presence of a single fluid or binary mixture. However, when a rotating porous matrix is considered, an additional body force exists in the form of the centrifugal acceleration. This force may generate free convection in the same manner as the gravity force causes natural convection. Vadasz [1] presented an analytical solution to the three-dimensional free convection problem in a long rotating porous box by using an asymptotic expansion method. The free convection resulted there from differential heating of the horizontal walls leading to temperature gradients orthogonal to the centrifugal body force. Secondary circulation was obtained in a plane orthogonal to the leading free convection plane as a result of the Coriolis effect on the flow.

This paper presents an analytical investigation of the Coriolis effect on free convection in a long rotating porous box subject to uniform heat generation. The volumetrically uniform heat generation introduces temperature gradients orthogonal to the centrifugal body force while two sets of thermal boundary conditions are considered for the top and bottom walls of the box. The first set considers both walls to be perfectly conducting, i.e. the same value of temperature is imposed on them. The second set considers the top wall to be perfectly conducting and the bottom wall perfectly insulated.

2. PROBLEM FORMULATION

A long rotating fluid saturated porous box subject to uniform heat generation is considered (Fig. 1). At each point of the flow domain the temperatures of the solid and fluid phases are assumed to be equal (Dagan [8]). The front, back and the lateral walls are all insulated. The box has a square cross section of which height and width is H_*, while the subscript * stands for dimensional values. The aspect ratio is defined as $a = H_*/L_*$ where L_* is the length of the box.

Given $\dot{Q}_*$ as the internal rate of heat generation and T_0 as a reference temperature value (e.g. the temperature imposed on the horizontal boundary) the dimensionless temperature can be represented by $T = (T_* - T_0)/(\dot{Q}_* H_*^2/\lambda_{e*})$ where λ_{e*} is the effective thermal conductivity of the porous domain and is assumed constant. Free convection occurs as a result of the centrifugal body force while the gravity force is neglected. The only inertial effects considered are the centrifugal acceleration, as far as changes in density are concerned, and the Coriolis force. Other than that the Darcy's law is assumed to govern the fluid flow (extended to include the centrifugal and Coriolis accelerations), while the Boussinesq approximation is applied for the effects of density variations. As the width (or height) of the domain is much smaller than its length (a small aspect ratio), a Cartesian coordinate

NOMENCLATURE

a	the aspect ratio of the box, equals H_*/L_*
Ek	Ekman number, equals $\nu_0\phi/2\omega_c k_*$
$\hat{\mathbf{e}}_x$	unit vector in the x direction
$\hat{\mathbf{e}}_y$	unit vector in the y direction
$\hat{\mathbf{e}}_z$	unit vector in the z direction
$\hat{\mathbf{e}}_n$	unit vector normal to the boundary, positive outwards
H_*	the height (width) of the box
k_*	permeability of the porous domain
L_*	the length of the porous domain
M_f	a ratio between the heat capacity of the fluid and the effective heat capacity of the porous domain
p	reduced pressure generalized to include the constant component of the centrifugal term (dimensionless)
$\mathbf{q}$	dimensionless specific flowrate vector equals $u\hat{\mathbf{e}}_x+v\hat{\mathbf{e}}_y+w\hat{\mathbf{e}}_z$
$\dot{Q}_*$	internal rate of heat generation
Ra_ω	porous media Rayleigh number related to the centrifugal body force, equals $\beta_*\dot{Q}_*H_*^3\omega_c^2L_*k_*M_f/\lambda_{e*}\alpha_{e*}\nu_*$
T	dimensionless temperature, equals $(T_*-T_0)\lambda_{e*}/\dot{Q}_*H_*^2$
T_0	a reference temperature value.
u	horizontal x component of the specific flowrate
v	horizontal y component of the specific flowrate
w	vertical component of the specific flowrate
x	horizontal length coordinate
y	horizontal width coordinate
z	vertical coordinate.

Greek symbols

ϕ	porosity
α_{e*}	effective thermal diffusivity
β_*	thermal expansion coefficient
ω_c	angular velocity of the rotating box
ν_*	fluid's kinematic viscosity
μ_*	fluid's dynamic viscosity
ψ	stream function
σ	a dimensionless group, equals Ra_ω/Ek.

Subscripts

*	dimensional values
0	zeroth order
1	first order.

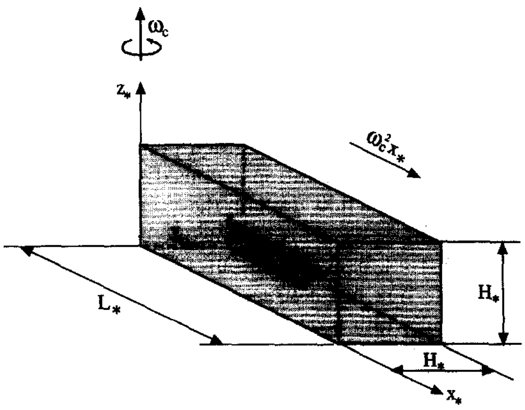

Fig. 1. A long rotating fluid saturated porous box subject to uniform heat generation.

system can be used and the component of the centrifugal acceleration in the y direction can be neglected. The ratio between the centrifugal force component in the y-direction and the corresponding component in the x-direction is $\omega_c^2y_*/\omega_c^2x_* = yH_*/xL_* = (y/x)a$. Therefore for length scales in the x-direction higher than the width of the box $(y/x)a \ll 1$, justifying to neglect the y-component of the centrifugal force. By assuming steady state conditions the following dimensionless set of governing equations is obtained

$$a\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$u = -\frac{\partial p}{\partial x} - Ra_\omega xT + Ek^{-1}v \tag{2}$$

$$av = -\frac{\partial p}{\partial y} - aEk^{-1}u \tag{3}$$

$$aw = -\frac{\partial p}{\partial z} \tag{4}$$

$$a^2\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}-au\frac{\partial T}{\partial x}-v\frac{\partial T}{\partial y}-w\frac{\partial T}{\partial z}+1=0. \tag{5}$$

Equations (1)–(5) are presented in a dimensionless form. The values α_{e*}/H_*M_f, $\mu_*\alpha_{e*}/k_*M_fa$, $\dot{Q}_*H_*^2/\lambda_{e*}$ and $\dot{Q}_*$ are used to scale the specific flowrate components (u_*, v_*, w_*), pressure (p_*), temperature variations (T_*-T_0) and the rate of heat generation, respectively, where α_{e*} is the effective thermal diffusivity, μ_* is the dynamic viscosity, k_* is the permeability of the porous matrix and M_f is the ratio between the heat capacity of the fluid and the effective heat capacity of the porous domain. Two different length scales were applied for scaling the variables x_*, y_* and z_*. Accordingly, $x = x_*/L_*$, $y = y_*/H_*$ and $z = z_*/H_*$. In equations (2) and (3) Ra_ω is the Rayleigh number modified to include the centrifugal body force instead of gravity in the form $Ra_\omega = \beta_*\dot{Q}_*H_*^3\omega_c^2L_*k_*M_f/\lambda_{e*}\alpha_{e*}\nu_*$ and Ek stands for the porous media Ekman number defined by $Ek = \nu_*\phi/2\omega_c k_*$ where ϕ is porosity, ω_c is the angu-