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Non-parallel thermal instability of forced convection flow over a heated, non-isothermal horizontal flat plate

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Abstract—A linear, non-parallel flow model is employed to study the onset of longitudinal vortex instability in laminar forced convection flow over a heated horizontal flat plate with variable surface temperature, $T_w(x) - T_\infty = Ax^n$. In the analysis, the streamwise dependence of the disturbance amplitude functions is taken into account. The resulting system of linearized disturbance equations for the amplitude functions constitutes an eigenvalue problem which is solved by a finite difference scheme along with Müller's shooting method. Neutral stability curves as well as the critical values for $Gr_x/Re_x^{3/2}$ and the corresponding critical wave numbers α^* are presented for Prandtl numbers $0.7 \leqslant Pr \leqslant 10^4$ over a range of the exponent $-0.5 \leqslant n \leqslant 1.0$. For a given Prandtl number, thermal instability is found to decrease as the value of the exponent n increases. Also, for a given value of the exponent n, fluids with larger Prandtl numbers are found to exhibit less susceptibility to instability than fluids with lower Prandtl numbers. However, this latter trend exists for $Pr \leqslant 100$. For $Pr > 100$, the critical values of $Gr_x/Re_x^{3/2}$ become essentially constant and independent of the Prandtl number. The results from the present non-parallel flow analysis are also compared with available analytical and experimental results from previous studies. The non-parallel flow analysis that accounts for the streamwise dependence of the amplitude functions is found to have a stabilizing effect as compared to the parallel flow analysis in which streamwise dependence of the disturbance is neglected.

INTRODUCTION

THE INSTABILITY of laminar boundary layer flows, due to either the wave mode or the vortex mode of instability, has been the subject of many studies. Instability of laminar forced convection flow over a horizontal, upward-facing heated plate, arising from the vortex mode of disturbance, was first analyzed by Wu and Cheng [1] using the linear stability theory. Moutsoglou *et al.* [2] also employed the linear theory to analyze the thermal instability of laminar mixed convection flow over a heated horizontal flat plate. In the latter study, the main flow and thermal fields were treated as non-parallel, but the disturbances were assumed to have the form of a stationary longitudinal vortex roll that is periodic in the spanwise direction. That is, the x-dependence of the disturbances was neglected. Recently, Chen and Chen [3] studied the vortex instability of laminar boundary layer flow over wedges by employing a non-parallel flow model that accounted for the streamwise dependence of the disturbances. Very recently, in re-examining the vortex instability of laminar forced convection flow over a heated horizontal flat plate, Yoo *et al.* [4] also considered the streamwise dependence of the disturbances and used the thermal boundary layer thickness as the reference length scale. Their analysis is good only for fluids with very large Prandtl numbers ($Pr \to \infty$) and solutions were obtained by power series. For this reason, their results are of limited practical utility.

In the present paper, vortex instability of laminar forced convection flow over a horizontal, upward-facing heated plate with the power-law variation in the surface temperature, $T_w(x) = T_\infty + Ax^n$, is examined for a wide range of Prandtl numbers, using the non-parallel flow model. The resulting eigenvalue problem for the disturbance amplitude functions was solved by an efficient finite difference method [5] in conjunction with Müller's shooting procedure.

Neutral stability curves as well as the critical values of $Gr_x/Re_x^{3/2}$ and the associated critical wave numbers were obtained for Prandtl numbers of 0.7, 7, 10^2, 10^3 and 10^4 over a range of the exponent values $-0.5 \leqslant n \leqslant 1.0$.

ANALYSIS

The main flow and thermal fields

As the first step in the analysis of the vortex instability of the flow, attention is directed to the main flow and thermal fields. Consider laminar forced convection over a horizontal flat plate with its heated surface facing upward and its surface temperature varying as $T_w(x) = T_\infty + Ax^n$, where A and n are real constants and T_∞ is the free stream temperature. The free stream velocity is U_∞. The physical coordinates are chosen such that x measures the streamwise distance from the leading edge of the plate and y the distance normal to the plate. Under the assumption of constant fluid properties, the transformed system of the boundary layer equations governing the main

NOMENCLATURE

a dimensionless wave number of disturbances
C_{fx} local friction factor, $\tau_w/(\rho U_\infty^2/2)$
f reduced stream function, $\psi/(\nu U_\infty x)^{1/2}$
g gravitational acceleration
Gr_x local Grashof number, $g\beta[T_w(x)-T_\infty]x^3/\nu^2$
Gr_L Grashof number based on L, $g\beta[T_w(L)-T_\infty]L^3/\nu^2$
L characteristic length
n exponent in the power-law variation of the wall temperature
Nu_x local Nusselt number
p' perturbation pressure
P main flow pressure
Pr Prandtl number
Re_x local Reynolds number, $U_\infty x/\nu$
Re_L Reynolds number based on L, $U_\infty L/\nu$
t dimensionless amplitude function of temperature disturbance
t' perturbation temperature
T main flow temperature
u, v, w dimensionless amplitude functions of velocity disturbance in the x, y, z directions, respectively
u', v', w' streamwise, normal, and spanwise components of perturbation velocity
U, V streamwise and normal velocity components of main flow in the x, y directions, respectively
x, y, z streamwise, normal, and spanwise coordinates
X, Y, Z dimensionless streamwise, normal, and spanwise coordinates, defined, respectively, as x/L, $y/\varepsilon L$, and $z/\varepsilon L$.

Greek symbols
α dimensionless wave number of disturbances, $aX^{1/2}$
$\tilde{\alpha}$ dimensionless wave number of disturbances based on thermal boundary layer thickness δ_t
β volumetric coefficient of thermal expansion
δ_m integral momentum boundary layer thickness
δ_t thermal boundary layer thickness
ε dimensionless parameter, defined as $Re_L^{-1/2}$
η similarity variable, $y(U_\infty/\nu x)^{1/2}$
θ dimensionless temperature, $(T-T_\infty)/[T_w(x)-T_\infty]$
κ thermal diffusivity of fluid
ν kinematic viscosity of fluid
ρ density of fluid
τ_w local wall shear stress
ψ stream function.

Superscripts
$+$ dimensionless disturbance quantity
$-$ scale quantity defined by equation (20)
$*$ critical condition or dimensionless main flow quantity
$\hat{}$ resultant quantity.

Subscripts
w condition at the wall
0 dimensionless amplitude function
∞ condition at the free stream.

flow and thermal fields can be expressed in dimensionless form as

$$f'''+\tfrac{1}{2}ff''=0 \quad (1)$$

$$\theta''+\tfrac{1}{2}Pr\,f\theta'-n\,Pr\,f'\theta=0 \quad (2)$$

$$f(0)=f'(0)=\theta(\infty)=0,\quad f'(\infty)=\theta(0)=1 \quad (3)$$

where the similarity variable $\eta(x, y)$, the reduced stream function $f(\eta)$, and the dimensionless temperature $\theta(\eta)$ are defined, respectively, as

$$\eta=y(U_\infty/\nu x)^{1/2},\quad f(\eta)=\psi/(\nu U_\infty x)^{1/2},$$
$$\theta(\eta)=(T-T_\infty)/[T_w(x)-T_\infty]. \quad (4)$$

In equations (1)–(3), the primes denote derivatives with respect to η and Pr is the Prandtl number. Other notations are as defined in the Nomenclature.

Equations (1)–(3) were solved by a finite difference method in conjunction with the cubic spline interpolation scheme to provide the main flow quantities that are needed in the thermal instability calculations and to provide other physical quantities, such as the axial velocity profile $f'(\eta)=U/U_\infty$, the temperature profile $\theta(\eta)$, the local Nusselt number Nu_x, and the local friction factor C_{fx}. In terms of the dimensionless variables, the last two quantities can be expressed, respectively, by

$$Nu_x\,Re_x^{-1/2}=-\theta'(0),\quad C_{fx}\,Re_x^{1/2}=2f''(0). \quad (5)$$

Formulation of the stability problem

In the present study, the linear stability theory is employed in the analysis. In experiments [6–9] the secondary flow vortex rolls have been found to be periodic in the spanwise direction. Thus, the disturbance quantities for velocity components u', v', w', pressure p', and temperature t' are assumed to be functions of (x, y, z). These disturbance quantities are superimposed on the two-dimensional main flow