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A mechanistic model of bubble entrainment in turbulent free surface flows

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ABSTRACT

A framework for the development of models of bubble entrainment in free surface turbulent flows is presented. The framework uses mechanistic processes to model the stages involved in the entrainment of bubbles due to interaction of turbulence with a free surface. Entrainment is modeled as a chain of events for a bubble that is formed at the free surface, then pulled into the fluid against buoyancy, and interacts with vortices that break it up into smaller bubbles, or with other bubbles by coalescence. The main entrainment mechanism is modeled as a vortex/free surface interaction process that can entrain bubbles if the vortices located a given distance from the surface are strong enough. This approach overcomes limitations of approaches where the entrainment is determined only by turbulence parameters, which in the case of objects interacting with a free surface entrain bubbles on the boundary layers irrespective of the distance to the free surface. Depending on the computational fluid dynamics approach used to solve the flow, these processes may need different levels of modeling; more resolved approaches like large-eddy simulation with a volume of fluid method of the free surface will require less modeling complexity than a less resolved RANS method, since some of the involved processes of entrainment are directly accounted for. In this paper a standard RANS approach is used with the free surface modeled using a single-phase level set method, and models are presented for each of the relevant processes to produce a complete mechanistic model of turbulent bubble entrainment. The model was calibrated and tested for two relevant problems: a 2D + T breaking wave in model scale, and the full scale bubbly flows around the US Navy Research Vessel Athena. For the second case a grid study is carried out to analyze grid convergence performance of the model. Comparisons with experimental data show that the model predicts well location and magnitude of entrainment.

1. Introduction

Bubble entrainment in liquid flows is of importance for a variety of applications, both naturally occurring and as result of human action. The entrainment of bubbles brings oxygen to rivers and lakes, improving water quality and making healthy aquatic life possible. On the other hand, an excess of bubbles at depth can increase the concentration of nitrogen in water, causing excess of total dissolved gas and bubble fish disease (Politano et al., 2009). Aeration also plays an important role in the dynamics of oceanic oxygenation (Deane and Stokes, 1999; Callaghan et al., 2014) and in the mixing of rivers carrying domestic and industrial effluents with lake and coastal waters (Chen and Jirka, 1999; Baddour et al., 2006). Bubble entrainment is frequently an undesired side effect affecting the performance of hydraulic structures (Odgaard, 1986; Ervine, 1998), ship propellers (Walker and Johnson, 1991; Anthony and Willmarth, 1992) and precipitation reactors (Rousseaux et al., 2000; Assirelli et al., 2008). In other applications the controlled entrainment of bubbles may be beneficial, such as in already mentioned oxygenation situations or ship drag reduction (Kumagai et al., 2015).

Though air/water is the prevalent case, bubble entrainment can occur in other gas/liquid systems, being the cause of impurities and defects in the pouring of molten metal (Zhang and Thomas, 2003) and of performance issues in pool type liquid metal fast breeder reactors (LMFBR) where argon bubbles are entrained by the turbulent liquid sodium (Patwardhan et al., 2012). In this paper the air/water system is used as a generic for gas/liquid, though the entrainment model can in principle be used for any gas/liquid system.

Entrapment of air into the water causes bubble entrainment, and occurs through a variety of processes, including low pressure regions, such as vortex cores and trailing edges of appendages,

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Main symbols

A	Transversal area
c_L, c_η, C	Turbulence spectrum constant
D	Bubble diameter
D_0	Formation bubble diameter
D_s	Diameter of ring-shaped deformation induced at the free surface by a single vortex
$\tilde{D}_s$	Non-dimensional form of D_s
D_H	Hinze scale
e	Eddy kinetic energy content
f	Bubble number density distribution
g	Gravity acceleration
J	Air volumetric flux per unit area
k_t	Turbulent kinetic energy
$\hat{k}$	Unit vector in vertical direction
ℓ	Vortex length scale
L	Characteristic length scale from k_t and ε_t
L_{11}	Integral length scale
L_{EI}	Transition length from the energy containing range to inertial range
L_{ID}	Transition length from the inertial range to the dissipation range
m	Bubble mass
$\boldsymbol{M}$	Interfacial force
$\hat{\boldsymbol{n}}$	Free surface normal pointing into the water
n_ℓ	Vortex number density
N	Bubble number density for a group
p	Pressure; probability function
p_0	Bubble size distribution at formation; constant in turbulence spectrum function
p_a	Atmospheric pressure
p_e	Entrainment size distribution
p_h	History bubble size distribution due to breakup and coalescence
p_z	Probability of finding a bubble at depth z
P_B	Probability of breakage
$\dot{Q}$	Single-vortex bubble entrainment rate
R	Bubble radius
s	Deformation slope of the free surface
$\tilde{s}$	Modified slope
$\mathbf{S}$	Strain rate tensor
S	Bubble entrainment source
$S_{\ell 0}$	Turbulent air entrainment constant
t	Time
Δt	Time step interval
$\boldsymbol{u}$	Velocity vector
u	RMS of velocity fluctuation in turbulence
$\bar{v}_0$	Mean bubble volume at formation
V_T	Bubble terminal velocity
dV, dV'	Differential volume
$\mathcal{V}$	Single vortex configuration
x	Coordinate in x direction
$\tilde{x}$	Non-dimensional x coordinate
y	Coordinate in y direction
y^+	Viscous dimensionless wall distance
z, z'	Depth under free surface
z_m	Depth for a vortex to induce a 45° slope on the free surface
z_e	For a single vortex, onset depth for entrainment
dz, dz'	Differential depth

Dimensionless groups

Fr	Froude number
We	Weber number
Fr_ℓ	Vortex Froude number
Re_λ	Taylor microscale Reynolds number
Sc	Bubble Schmidt number

Greek letters

α	Volume fraction
β	Bubble breakup source; turbulence modeling constant
δ	Surface roughness
ε_t	Turbulence dissipation
η	Kolmogorov length scale
κ	Wave number
λ	Mixing length scale
μ	Dynamic viscosity
μ_{eff}	Effective dynamic viscosity
ν	Kinematic viscosity
ν_{t0}	Turbulent viscosity at the free surface
ϕ	Signed distance to the free surface (level set function)
ρ	Mass density
σ	Surface tension
χ	Bubble coalescence source

Subscripts

d	Disperse phase
c	Continuous phase
cr	Critical value
g	Group number
i, j	Indices from 1 to 3
m	Bubbles with mass m
min	Minimum
M	Maximum
s	Free surface
λ	Mixing length model with length scale λ
ℓ	Vortex with size ℓ

collapse of air films caused by plunging jets. In these cases bubbles are entrained into the water when drag from the interface into the liquid overcomes buoyancy and surface tension forces that usually resist entrainment. Direct entrapment can also occur through collapse of cavities formed by plunging breaking waves, by drop impact, etc. The entrapped air is subject to the simultaneous action of breakup and coalescence, caused by turbulence, shear and buoyancy, resulting in a dynamic bubble size distribution. These various mechanisms obey completely different physics and should thus be modeled independently. For instance, in the case of a surface ship bow and shoulder waves often exhibit plunging jets and high levels of turbulence. The contact line between the hull and the free surface is subject to the highly turbulent ship boundary layer. The stern flow, mostly in transom sterns at lower Froude numbers, exhibits highly energetic large-scale vortices and massive entrainment.

Though computers and numerical algorithms have made enormous progress in recent years, direct simulation of bubble entrainment in full scale applications is still many decades off. Fig. 1 shows a diagram of the scales involved in bubble entrainment problems, where the largest scales are related to the size of the application and shown in the order of a hundred meters, representative for example of a nuclear reactor, a spillway or a ship. The smallest scales are related to the processes of bubble coalescence, involving film breakup processes and retarded Van der Waals forces where scales as small as 0.01 μm are of importance. Bubbles and air cavities can have sizes between 20 μm to 1 cm. Consider the direct numerical simulation of a large scale problem