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New dimensionless number for gas–liquid flow in pipes

Alsarkhi Abdelsalam[a,*], Sarica Cem[b], Pereyra Eduardo[b]

[a] Department of Mechanical Engineering, King Fahd University of Petroleum & Minerals, Saudi Arabia
[b] Department of Petroleum Engineering, The University of Tulsa, Tulsa, OK, USA

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ABSTRACT

Two-phase flow modeling is a general problem in science and engineering. Two-phase flow phenomenon is inherently complicated and characterized by a large number of flow variables. It is historically known that the lack of proper dimensionless numbers in two-phase flow is one of the major shortcomings as compared to single-phase flow. A new dimensionless number (Slippage Number) for gas–liquid flow in pipes is proposed in this paper. The number is defined as the ratio of the difference in the gravitational forces between slip and no-slip conditions to the inertial force of the gas. It is found to be a function of Froude number based on the mixture velocity especially in the elongated bubble, slug, churn, bubble, and high film thickness wavy annular flow patterns. The liquid holdup data for a wide range of fluid and flow conditions (different viscosities, densities, pipe diameters, inclination angles, gas and liquid flow rates) can be correlated with a single curve using the Slippage Number. The value of the number varies from highest to lowest for bubble, elongated bubble, slug, churn, stratified and annular flow patterns, respectively. It is close to zero for homogeneous flow patterns like mist and dispersed bubble flows. We also show that this number may be used as a flow pattern identifier.

Discussion on currently available relevant dimensionless numbers

A survey of related (or somehow close in mathematical formulation to the proposed one) dimensionless numbers are presented in this section. The closest dimensionless number to the proposed one is the Richardson Number, *Ri*. *Ri* is defined as the ratio of the buoyancy force to inertial force (Eq. (1)). It can also be found by multiplying the density ratio, $\Delta\rho/\rho$, with the Froude number, *Fr*. Sometimes, it is even called Froude number, *Fr*.

$$Ri = \frac{(\Delta p)gD}{pv^2} \tag{1}$$

Ri is used in a single-phase flow in pipes when there is a change in the density, or in multiphase flow in pipes when there is a gas bubble in liquid in which the density ratio is represented by the difference between the gas and liquid densities to the liquid density. If *Ri* is very close to one, then the flow is most likely to be buoyancy-driven flow.

Sometimes, *Ri* is presented as the ratio of the buoyancy term to the flow gradient term as: (Eq. (2))

$$Ri = \frac{(\nabla p)g}{p(\nabla v)^2} \tag{2}$$

* Corresponding author. Tel.: +966 138607725.
E-mail address: alsarkhi@kfupm.edu.sa (A. Abdelsalam).

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It is of practical importance in weather forecasting and in investigating density and cloudiness or haziness of liquid currents in oceans, rivers, lakes, and reservoirs. The cloudiness or haziness sometimes called turbidity, which is a test of water quality due to presence of invisible particles to the naked eye.

In aviation, *Ri* is a measure of kinetic energy, and it measures air turbulence possibilities. A lower value of *Ri* means more severe turbulence, and a value between 10 and 0.1 is typical. Along the same line as *Ri* is what so-called Wedderburn number, W, (see Shintani et al. 2010). In Wedderburn number the used density ratio is the difference between the density of the upper and lower layers fluid to the lower layer fluid density.

The other dimensionless number, which is close to the proposed one, is Froude Number, *Fr*. It is defined as the ratio of the inertial force to the gravitational force as given in Eq. 3.

$$Fr = \frac{v^2}{gD} \tag{3}$$

Different forms of Froude number can be found in the literature. The most known are the ones used by Lockhart and Martinelli (1949) as given by Eqs. (4) and (5) for liquid and gas phases, respectively.

$$Fr_l = \sqrt{\frac{p_l v_{SL}^2}{(p_l - p_g)gD\cos\theta}} \tag{4}$$

$$Fr_g = \sqrt{\frac{p_g v_{SG}^2}{(p_l - p_g) g D \cos\theta}} \tag{5}$$

The new dimensionless number (Slippage Number, SL)

In gas–liquid flow, the average two-phase flow mixture density or the slip-density, ρ_{TP}, is different from the homogenous or no-slip density of the mixture, ρ_H, due to slippage between phases. The no-slip mixture density, ρ_H, is simply calculated based on the ratio of the volume flow rate of the phases assuming no slippage between the phases. The slip density, ρ_{TP}, is calculated based on the actual or measured liquid holdup. The proposed dimensionless number is defined as the ratio of the difference in the gravitational forces between slip and no-slip conditions to the inertial force of the gas (based on the superficial gas velocity or the volumetric flux of the gas) as given by Eq. (6).

$$SL = \frac{(p_{TP} - p_H) g D}{p v_{SG}^2} \tag{6}$$

The average two-phase flow density based on the in-situ liquid holdup and the average two-phase density based on homogeneous (no-slip) holdup or liquid volume fraction are defined in Eqs. (7) and (8), respectively.

$$p_{TP} = p_L H_L + (1 - H_L) p_G \tag{7}$$

$$\rho_H = \lambda_L \beta_L + (1 - \lambda_L) \rho_G \tag{8}$$

Above, ρ_G, ρ_L, H_L, and λ_L, are: gas density, liquid density, actual liquid holdup, and liquid volume fraction, respectively. λ_L is defined by Eq. (9)

$$\lambda_L = \frac{v_{SL}}{v_{SL} + v_{SG}} = \frac{v_{SL}}{v_M}, \tag{9}$$

where, v_{SL}, v_{SG}, and V_M are superficial liquid velocity, superficial gas velocity, and mixture velocity, given with Eqs. (10) and (11), respectively.

$$v_{SL} = \frac{W_L}{\rho_L A_P}, \tag{10}$$

$$v_{SG} = \frac{W_G}{\rho_G A_P}, \tag{11}$$

where, W_L and W_G are liquid and gas mass flow rates, respectively.

The value of Slippage Number, *SL*, for a given pipe diameter and fluids depends on the superficial gas velocity and the difference between the mixture densities. The difference in the mixture densities is very much function of the flow pattern and the slippage between the phases. For example, for bubbly flow typically experienced at low gas superficial velocities, the number will be high. However, for annular or mist flow at high superficial gas velocities, the value of the number will be very low since the slippage is minimum, and the liquid droplets are carried by gas, especially, for low v_{SL} or small liquid film thicknesses. Slug flow in a vertical pipe involves falling film and significant slippage between the phases. Thus, the difference between the slip and no-slip densities is large resulting in a relatively large Slippage Number. Churn flow in vertical pipe can be observed at higher gas velocities than those of slug flow. Thus, the value of this number is less than that in slug flow. The elongated-bubble flow is considered to be a simple case of slug flow when the liquid slug is free of entrained bubbles. This occurs at relatively lower gas rates compared to slug flow. Thus, the slippage between gas and liquid expected to be larger than slug flow, and hence, the difference between the two densities is larger as well as the Slippage Number. Dispersed bubble flow has much less slippage between phases in the pipe resulting in Slippage Number very close to zero. Therefore, this number can be used as a quick flow pattern identification method. Moreover, it can be very useful in modeling of gas–liquid flows in pipes, especially in drift flux approach, since the drift flux approach is based on the slippage between the phases.

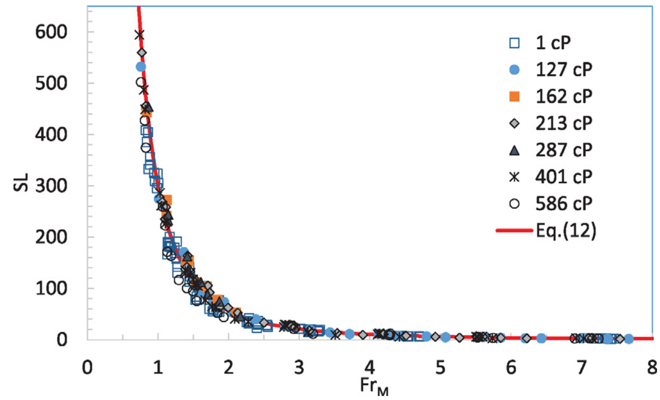

Fig. 1.A. Slippage Number versus Froude number based on mixture velocity for Alruhaimani (2015) experimental data.

Results

Initially, the relationship between the Slippage Number, *SL*, and the mixture Froude number, Fr_M, is demonstrated in this section using recent experimental data of Alruhaimani (2015). The mixture Froude number, Fr_M, is defined as in Eq (12)

$$Fr_M = \sqrt{\frac{\rho_L}{\rho_L - \rho_G}} \frac{v_M}{\sqrt{gD}} \tag{12}$$

Then, the results of nine different experimental data sets covering wide range of fluids properties, pipe diameter, inclination angles and operational conditions with different flow patterns are presented. Dispersed and mist flow patterns are not considered since, in both flows, the phases are homogeneously mixed and flow with the same velocity with negligible slippage.

High viscosity oil and air flow in upward vertical pipes (Alruhaimani, 2015)

Alruhaimani (2015) experimental data for air–oil flow in a vertical 0.0508 m diameter pipe are used to validate the proposed relation between *SL* and Fr_M. Tests were conducted for various oil viscosities, namely, 586, 401, 287, 213, 162, and 127 mPa·s. The superficial liquid and gas velocities were varied from 0.05 m/s to 0.7 m/s and from 0.5 m/s to 5 m/s, respectively. The flow patterns observed were slug, churn and annular flows.

Figs. 1.A. and 1.B. show *SL* vs. Fr_M in Cartesian and Semi-log coordinate systems, respectively. As seen from the Fig., there is an excellent correlation between *SL* vs. Fr_M for all tested flow patterns (slug, churn flow and annular). Eq. (13) gives the correlation obtained through curve fitting. In addition to high viscosity oil and air tests, air–water experiments also have been conducted using the same facility. The air–water results plotted on the same plot shows an excellent agreement with high viscosity oil and air data.

$$SL = 300.13 Fr_M^{-2.425} \tag{13}$$

where Fr_M is the mixture Froude number.

In Fig. 1.C. the data were sorted based on the observed flow patterns. It can be seen that the *SL* varies from highest for slug flow to lowest for the annular flow. It is noted here that Alruhaimani (2015) could not visually or using the normalized voltage histogram from the capacitance sensors determine the type